Innovation, Networks, and Power

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Acknowledgments and sources

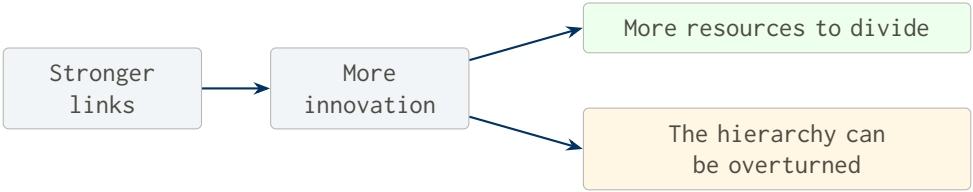
I thank **Federico Echenique** and **Luan Borelli**.

This paper draws on:

- **The Market and the Jungle** (Carbajal, Gallardo, and Martinelli, 2026)
- **Equilibrium in the Jungle** (Piccione and Rubinstein, 2007)
- **Acemoglu's Nobel lecture** (2025)
- **Geoeconomics:** Clayton, Maggiori, and Schreger.
  A Framework for Geoeconomics (2026);
  A Theory of Economic Coercion and Fragmentation (forthcoming).
- **Networks:** Jackson and Wolinsky (1996); Ballester, Calvó-Armengol, and Zenou (2006); Bloch and Dutta (2009); Hellmann (2013)

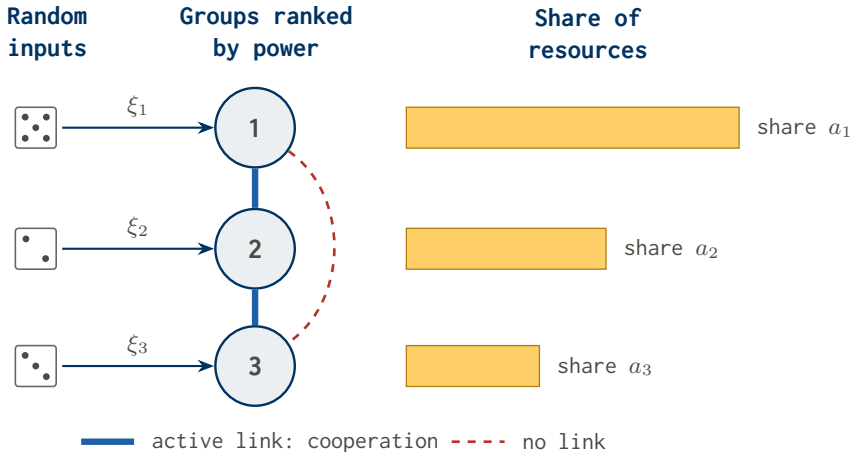
Low innovation, little cooperation

- **Latin America:** segregated societies, little innovation.
- Cooperation raises innovation but spreads the gains.



Question: does the fear of losing power stop groups from cooperating?

The setting in one picture



Links spread innovation. Innovation adds resources and can reorder the ranking.

This paper: model and equilibrium

- Groups are ranked by power and form links by mutual consent.
- Links raise innovation.
- Innovation expands resources and can reopen the ranking.

Equilibrium: pairwise stability

For every pair, with all other links fixed:

1. neither partner gains by weakening or cutting the link;
2. no higher link strength benefits one partner without hurting the other.

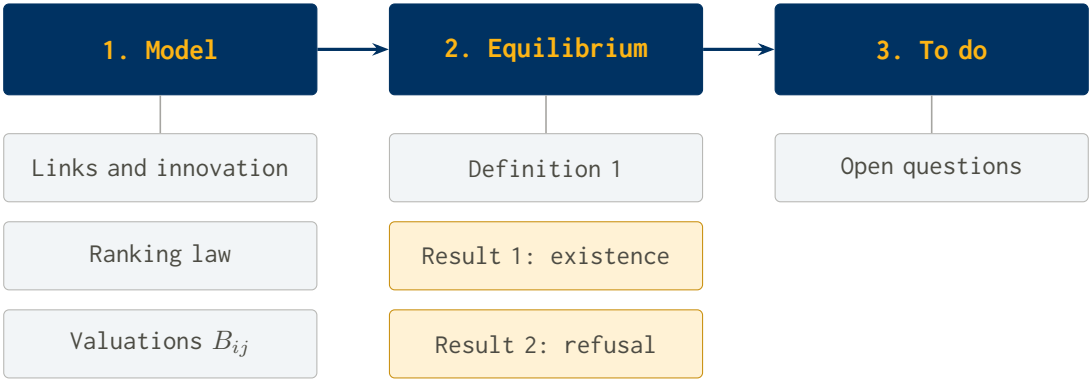
This paper: results

1. **Existence.** Under a condition on parameters only, a greatest stable network exists, and we can compute it.
2. **Refusal.** At high exposure, the leader rejects every incident link, even though it gains from its own innovation.

Withdrawal spillover: when the leader exits, third-party links can also collapse.



Roadmap



Gold boxes are the main results.

The model

Groups form links by mutual consent

- $N \geq 2$ groups of equal size, ranked $1 \succ_0 \cdots \succ_0 N$.
- Each partner proposes a link strength (ℓ_{ij}, ℓ_{ji}); the smaller proposal is realized:

$$\rho_{ij} = \min\{\ell_{ij}, \ell_{ji}\} \in \{0\} \cup [\underline{\rho}_{ij}, \overline{\rho}_{ij}], \qquad 0 < \underline{\rho}_{ij} \leq \overline{\rho}_{ij}.$$

- Links are costly:

$$\mathcal{C}_i(\boldsymbol{\rho}) = \sum_{j \neq i} \left[\kappa_0 \mathbf{1}\{\rho_{ij} > 0\} + \kappa_1 \rho_{ij}\right], \qquad \kappa_0 > 0, \;\; \kappa_1 \geq 0.$$

Innovation spreads through the network

Matrix form:

$$\mathbf{y} = \boldsymbol{\xi} + \beta\, \mathbf{G}(\boldsymbol{\rho})\, \mathbf{y}, \qquad \mathbf{G}(\boldsymbol{\rho}) = \left[\rho_{ij}/\bar{\rho}\right], \quad \bar{\rho} = \max_e \overline{\rho}_e, \;\; \rho_{ii} = 0.$$

For each group i:

$$\underbrace{y_i}_{\text{innovation}} = \underbrace{\xi_i}_{\text{own input}} + \underbrace{\beta \sum_{j \neq i} \frac{\rho_{ij}}{\bar{\rho}}\, y_j}_{\text{spillovers from partners}}$$

Inputs $\boldsymbol{\xi}$ are random: bounded, $0 \leq \boldsymbol{\xi} \leq \overline{\mathbf{v}}$, with mean $\mathbf{v} \gg 0$.

Innovation is well defined for every network

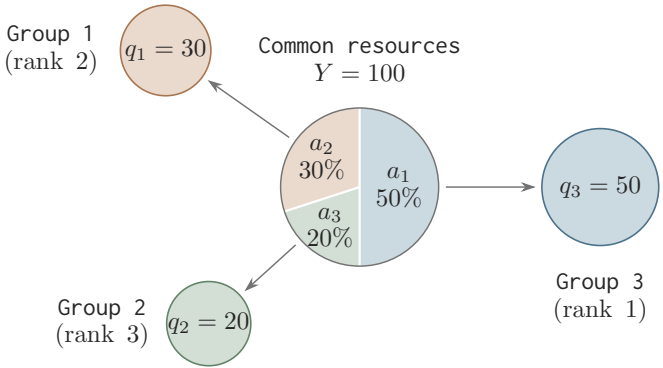
If $0 < \beta < 1/(N-1)$, the Neumann series converges

$$\mathbf{y} = \mathbf{M}(\boldsymbol{\rho})\boldsymbol{\xi}, \qquad \mathbf{M}(\boldsymbol{\rho}) = [\mathbf{I} - \beta\mathbf{G}(\boldsymbol{\rho})]^{-1} = \sum_{k \geq 0} \beta^k \mathbf{G}(\boldsymbol{\rho})^k.$$

Why: each row of $\beta\mathbf{G}(\boldsymbol{\rho})$ sums to at most $\beta(N-1) < 1$.

Rank divides resources

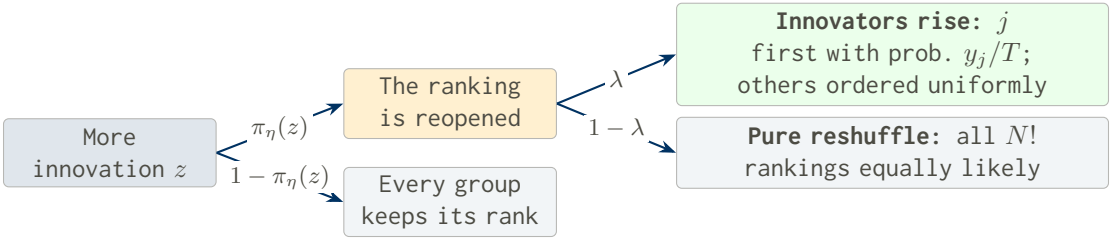
Resources are $Y(z) = Y_0 + Dz$ $(Y_0, D > 0)$, where z is average innovation. Rank r consumes share a_r $(a_1 \geq \cdots \geq a_N > 0,\ \sum_r a_r = 1,\ a_1 > 1/N)$.



Innovation is an opportunity to take power

Someone can appropriate the new resources and rise to the top.

When $\lambda > 0$, whoever created the innovation is more likely to be that someone.



$$\pi_\eta(z) = \frac{\eta z}{Y_0 + Dz}, \qquad \mathbb{P}(i \text{ first} \mid \text{reopened}) = \frac{1-\lambda}{N} + \lambda \frac{y_i}{T} \quad \big(T = \textstyle\sum_j y_j > 0\big).$$

Is $\pi_\eta(z)$ a probability? The bound $\overline{h}$

All links at capacity, all inputs at their maximum:

$$\overline{h} = \frac{1}{N}\,\mathbf{1}^\top \mathbf{M}(\boldsymbol{\rho}^{\max})\,\overline{\mathbf{v}}.$$

Claim: $z \le \overline{h}$ **always.** A larger network and larger inputs only raise innovation:

$$\mathbf{y} = \mathbf{M}(\boldsymbol{\rho})\,\boldsymbol{\xi} \;\le\; \mathbf{M}(\boldsymbol{\rho}^{\max})\,\boldsymbol{\xi} \;\le\; \mathbf{M}(\boldsymbol{\rho}^{\max})\,\overline{\mathbf{v}} \quad\Longrightarrow\quad z = \tfrac{1}{N}\mathbf{1}^\top\mathbf{y} \le \overline{h}.$$

So $\pi_\eta(z)$ is a probability whenever

$$0 \le \eta \le D + \frac{Y_0}{\overline{h}}$$

Proof: a larger network gives a larger $\mathbf{M}$

To show: $\boldsymbol{\rho} \leq \boldsymbol{\rho}^{\max}$ implies $\mathbf{M}(\boldsymbol{\rho}) \leq \mathbf{M}(\boldsymbol{\rho}^{\max})$, entry by entry.

1. Weaker links give a smaller network matrix: $0 \leq \mathbf{G}(\boldsymbol{\rho}) \leq \mathbf{G}(\boldsymbol{\rho}^{\max})$.
2. Powers of nonnegative matrices keep the order: $\mathbf{G}(\boldsymbol{\rho})^k \leq \mathbf{G}(\boldsymbol{\rho}^{\max})^k$ for every k.
3. Add up the Neumann series, with weights $\beta^k \geq 0$:

$$\mathbf{M}(\boldsymbol{\rho}) = \sum_{k \geq 0} \beta^k \mathbf{G}(\boldsymbol{\rho})^k \;\leq\; \sum_{k \geq 0} \beta^k \mathbf{G}(\boldsymbol{\rho}^{\max})^k = \mathbf{M}(\boldsymbol{\rho}^{\max}).$$

Inputs: since $\mathbf{M} \geq 0$ and $\boldsymbol{\xi} \leq \overline{\mathbf{v}}$, also $\mathbf{M}(\boldsymbol{\rho}^{\max})\,\boldsymbol{\xi} \leq \mathbf{M}(\boldsymbol{\rho}^{\max})\,\overline{\mathbf{v}}$.

Where does each group end up?

For $T > 0$, p_{ir} is the probability that **group i ends up in rank r** after the ranking is reopened:

$$p_{ir} = \begin{cases} \dfrac{1-\lambda}{N} + \lambda \dfrac{y_i}{T}, & r = 1, \\ \dfrac{1-\lambda}{N} + \dfrac{\lambda}{N-1}\left(1 - \dfrac{y_i}{T}\right), & r \geq 2. \end{cases}$$

- First place rewards innovation; lower ranks are exchangeable.
- **Proposition 1:** when λ is large enough, own innovation lowers an incumbent's probability of losing rank.

Example: three groups

Outputs $\mathbf{y} = (3, 2, 1)$ and $\lambda = 1$. Rows are groups, columns are ranks:

$$[p_{ir}] = \begin{pmatrix} 1/2 & 1/4 & 1/4 \\ 1/3 & 1/3 & 1/3 \\ 1/6 & 5/12 & 5/12 \end{pmatrix}$$

Group 1 produced half of total innovation: under a redraw, it takes first place with probability $1/2$.

Why not reward innovation at every rank?

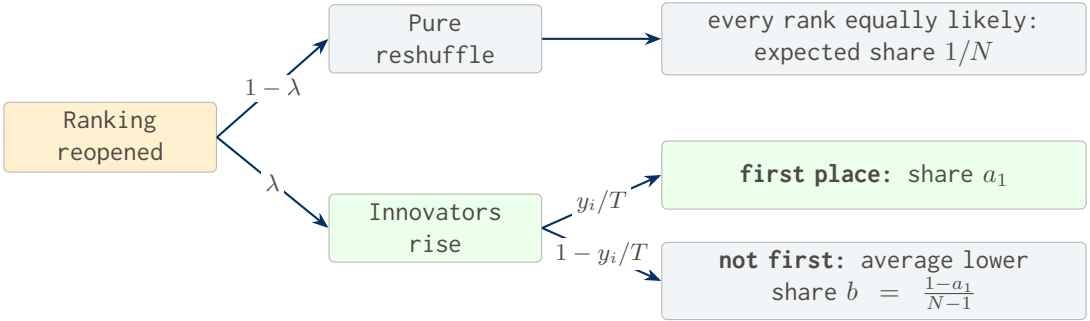
Sequential rule (Plackett–Luce): fill ranks from the top; at each step, pick a remaining group in proportion to its innovation.

$$P^{\mathrm{PL}}(\sigma \mid \mathbf{y}) = \prod_{r=1}^{N} \frac{y_{\sigma(r)}}{\sum_{s \geq r} y_{\sigma(s)}}$$

- **More natural:** innovation helps at every rank.
- **But** payoffs become nonlinear in innovation. Open question.

The expected share after a redraw (1): two draws

Suppose the ranking is reopened ($T > 0$). What share can group i expect?



The expected share after a redraw (2): add them up

Innovators rise:

$$\frac{y_i}{T}\,a_1 + \left(1 - \frac{y_i}{T}\right) b \;=\; b + \underbrace{(a_1 - b)}_{\Delta\text{: premium of first place}} \frac{y_i}{T}$$

Both draws together:

$$(1-\lambda)\,\frac{1}{N} + \lambda\left(b + \Delta\,\frac{y_i}{T}\right) \;=\; \underbrace{\frac{1-\lambda}{N} + \lambda\, b}_{m_\lambda\text{: expected share if } y_i = 0} + \lambda\,\Delta\,\frac{y_i}{T}$$

Where the payoff comes from

Consumption. Group i ends up in rank R_i and consumes that rank's share:

$$q_i = a_{R_i}\, Y(z).$$

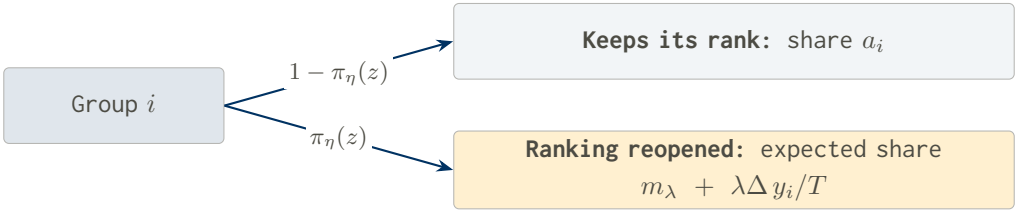
Payoff. Expected consumption minus the cost of its links:

$$U_i(\boldsymbol{\rho}) = \mathbb{E}[\, q_i \,] - C_i(\boldsymbol{\rho}).$$

Two things are random: **the ranking** and **the inputs**. We average over each in turn.

Averaging over the ranking (1): keep or redraw

Fix innovation $\mathbf{y}$. Group i starts in rank i:



Expected consumption is resources times the expected share:

$$\mathbb{E}[\, q_i \mid \mathbf{y}\,] = Y(z)\Big[\big(1-\pi_\eta(z)\big)\, a_i + \pi_\eta(z)\Big(m_\lambda + \lambda\Delta\,\frac{y_i}{T}\Big)\Big].$$

Averaging over the ranking (2): resources cancel

The key fact

$$\pi_\eta(z)\, Y(z) = \eta\, z, \qquad \text{because} \quad \pi_\eta(z) = \frac{\eta z}{Y(z)}.$$

Rearranging the previous slide:

$$\mathbb{E}[\, q_i \mid \mathbf{y}\,] = a_i\, Y(z) \;+\; \eta\, z \left(m_\lambda + \lambda\Delta\, \frac{y_i}{T} - a_i \right).$$

Total resources $Y(z)$ cancel out.

Averaging over the ranking (3): collect terms

Use $Y(z) = Y_0 + Dz$ and $z = T/N$:

$$\mathbb{E}[\, q_i \mid \mathbf{y} \,] = a_i Y_0 \;+\; D a_i \, z - \eta \, (a_i - m_\lambda) \, z + \frac{\eta \lambda \Delta}{N} \, y_i .$$

Since $z = \frac{1}{N} \sum_j y_j$, this is linear in every group's innovation:

$$\mathbb{E}[\, q_i \mid \mathbf{y} \,] = a_i Y_0 + \sum_j B_{ij} \, y_j , \qquad B_{ij} = \frac{1}{N} \Big[\, D a_i - \eta \, (a_i - m_\lambda) + \eta \lambda \Delta \, \mathbf{1}\{i = j\} \, \Big] .$$

What is innovation by group j worth to group i?

B_{ij} is the change in group i's expected consumption when group j produces one more unit of innovation. Each color has a meaning:

$$B_{ij} = \frac{1}{N}\Big[\, D a_i - \eta\,(a_i - m_\lambda) + \eta\lambda\Delta\,\mathbf{1}\{i = j\} \,\Big]$$

- **Bigger pie.** Status-quo share a_i of the new resources.
- **Risk of a redraw.** A loss when $a_i > m_\lambda$, a gain when $a_i < m_\lambda$.
- **Own-innovation premium.** Only if $j = i$: a better chance of first place.

Ex ante payoffs

Finally, average over the inputs:

$$U_i(\boldsymbol{\rho}) = a_i Y_0 + \underbrace{f_i(\boldsymbol{\rho})}_{\text{gross benefit of the network}} - \underbrace{\mathcal{C}_i(\boldsymbol{\rho})}_{\text{cost of links}}$$

$$f_i(\boldsymbol{\rho}) = \underbrace{B_{ii}\,\mathbb{E}[y_i]}_{\text{own innovation}} + \underbrace{\sum_{j \neq i} B_{ij}\,\mathbb{E}[y_j]}_{\text{others' innovation}}$$

- $a_i Y_0$: what group i consumes without any innovation.
- Whether i wants a link depends on the signs of B_{ii} and B_{ij} $(j \neq i)$.

Definition 1: pairwise stability

Let $\Gamma_k(e,t;\boldsymbol{\rho}) = U_k(\boldsymbol{\rho}^{e,t}) - U_k(\boldsymbol{\rho})$: the change in k's payoff when link e moves to strength t, all other links fixed.

Definition 1

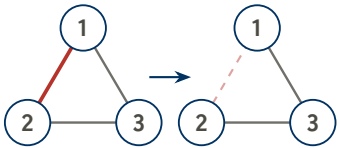
A feasible network $\boldsymbol{\rho}$ is **pairwise stable** if, for every pair $e = ij$:

1. **No unilateral reduction.** $\Gamma_i(e,t;\boldsymbol{\rho}) \leq 0$ and $\Gamma_j(e,t;\boldsymbol{\rho}) \leq 0$ for every feasible $t < \rho_e$.
2. **No bilateral increase.** No feasible $t > \rho_e$ has $\Gamma_i(e,t;\boldsymbol{\rho}) \geq 0$ and $\Gamma_j(e,t;\boldsymbol{\rho}) \geq 0$, with at least one strict.

Deviations are tested one link at a time, with all other links fixed.

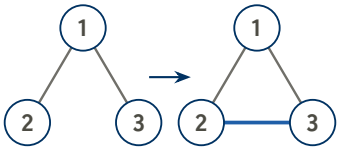
The three deviations a stable network must resist

Delete or weaken



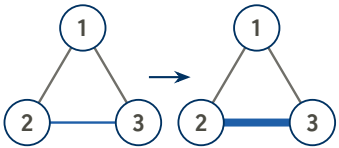
Group 1 cuts link 12.
Unilateral.

Form



Groups 2 and 3 add link 23.
Needs both.

Strengthen



Groups 2 and 3 raise ρ_{23}.
Needs both.

A network is pairwise stable when no pair has a profitable change of this kind.

The sign condition

For every group i, one of two cases:

- **Weakly gains from everyone's innovation:** $B_{ii} \geq 0$ and $B_{ij} \geq 0$ for all $j \neq i$.
- **Weakly loses from everyone's innovation:** $B_{ii} \leq 0$ and $B_{ij} \leq 0$ for all $j \neq i$.

What it rules out

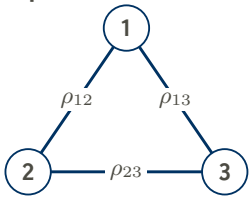
A group that gains from its own innovation and loses from others': $B_{ii} > 0 > B_{ij}$.

The B_{ij} are constants, so this is a condition on parameters only.

The network as a matrix: G

In general: $\mathbf{G}(\boldsymbol{\rho}) = \left[\rho_{kj}/\bar{\rho}\right]_{k,j}$, an $N \times N$ symmetric matrix with zero diagonal.

Three groups:



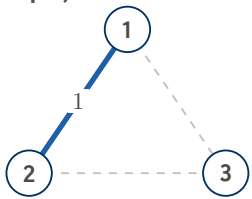
$$\mathbf{G}(\boldsymbol{\rho}) = \frac{1}{\bar{\rho}} \begin{pmatrix} 0 & \rho_{12} & \rho_{13} \\ \rho_{12} & 0 & \rho_{23} \\ \rho_{13} & \rho_{23} & 0 \end{pmatrix}$$

Entry (k, j) is the normalized strength of the link between k and j.

One link as a matrix: $\mathbf{S}_e$

In general: for the link $e = kj$, $\mathbf{S}_e$ has ones at positions (k, j) and (j, k), zeros elsewhere.

Three groups, link 12:

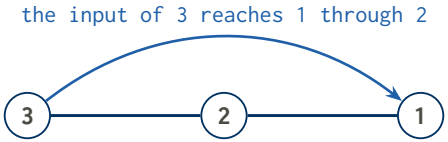


$$\mathbf{S}_{12} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Strengthening link e moves $\mathbf{G}$ in the direction $\mathbf{S}_e$.

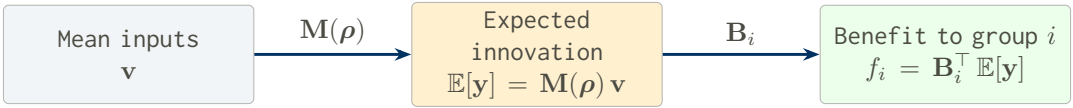
From inputs to innovation: $\mathbf{M}$

$$\mathbf{M}(\boldsymbol{\rho}) = [\mathbf{I} - \beta \mathbf{G}(\boldsymbol{\rho})]^{-1} = \underbrace{\mathbf{I}}_{\text{own input}} + \underbrace{\beta\,\mathbf{G}}_{\text{partners}} + \underbrace{\beta^2 \mathbf{G}^2}_{\text{partners of partners}} + \cdots$$



- Entry (k, j) of $\mathbf{M}$: total effect of group j's input on group k's innovation.
- Expected innovation of all groups: $\mathbb{E}[\mathbf{y}] = \mathbf{M}(\boldsymbol{\rho})\,\mathbf{v}$.

The benefit of the network: f_i



$$f_i(\boldsymbol{\rho}) = \sum_j B_{ij}\,\mathbb{E}[y_j] = \mathbf{B}_i^\top\,\mathbf{M}(\boldsymbol{\rho})\,\mathbf{v}$$

- $\mathbf{B}_i = (B_{i1}, \ldots, B_{iN})^\top$: what group i gets from each group's innovation.
- Links change f_i only by changing $\mathbf{M}(\boldsymbol{\rho})$.

Convexity

For a group i that gains from everyone's innovation ($\mathbf{B}_i \geq 0,\ B_{ii} > 0$):

Doubling an incident link's strength more than doubles the gain from that link.

Consequence under the sign condition

With a fixed activation cost $\kappa_0 > 0$, every active link in equilibrium operates at full capacity.

Convexity, in matrices

How does innovation react to a stronger link e?

$$\frac{\partial \mathbf{M}}{\partial \rho_e} = \frac{\beta}{\bar{\rho}} \underbrace{\mathbf{M}}_{\text{reach the link}} \underbrace{\mathbf{S}_e}_{\text{cross it}} \underbrace{\mathbf{M}}_{\text{spread again}} \geq 0$$

Do it twice:

$$\frac{\partial^2 \mathbf{M}}{\partial \rho_e^2} = \frac{2\beta^2}{\bar{\rho}^2} \mathbf{M}\mathbf{S}_e\mathbf{M}\mathbf{S}_e\mathbf{M} \geq 0$$

All entries are nonnegative. With $\mathbf{B}_i \geq 0$, $f_i = \mathbf{B}_i^\top \mathbf{M}\mathbf{v}$ is nondecreasing and convex in every link (strictly so on incident links when $B_{ii} > 0$).

Supermodularity

Definition. For a group with $\mathbf{B}_i \geq 0$, adding a link at capacity is worth more in a larger network. For active-link sets $A \subseteq B$ and $e \notin B$:

$$f_i(\boldsymbol{\rho}^{B \cup \{e\}}) - f_i(\boldsymbol{\rho}^{B}) \;\geq\; f_i(\boldsymbol{\rho}^{A \cup \{e\}}) - f_i(\boldsymbol{\rho}^{A}),$$

where $\boldsymbol{\rho}^{A}$ sets links in A at capacity and all others to zero.

What it means here

If a link does not pay in a large network, it does not pay in a smaller one. So a link that was deleted is never worth adding back.

Supermodularity, in matrices

How does link g change the effect of link e? Cross both, in either order:

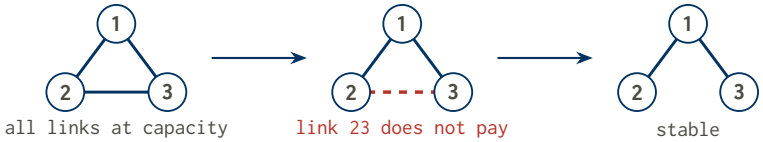
$$\frac{\partial^2 \mathbf{M}}{\partial \rho_e \, \partial \rho_g} = \frac{\beta^2}{\bar{\rho}^2} \Big(\underbrace{\mathbf{M}\,\mathbf{S}_e\,\mathbf{M}\,\mathbf{S}_g\,\mathbf{M}}_{\text{cross } e\text{, then } g} + \underbrace{\mathbf{M}\,\mathbf{S}_g\,\mathbf{M}\,\mathbf{S}_e\,\mathbf{M}}_{\text{cross } g\text{, then } e} \Big) \;\geq\; 0$$

All entries are nonnegative. With $\mathbf{B}_i \geq 0$, a stronger link g raises the marginal benefit of link e.

Result 1: a greatest stable network exists

Under the sign condition:

1. Isolate groups with $\mathbf{B}_i \leq 0$; put all remaining links at full capacity.
2. Delete any link that fails to cover its cost for at least one partner.
3. Repeat until no such link remains.



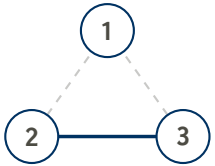
Theorem 1

Every stable network is saturated. The procedure ends at the **greatest** pairwise stable network (containing all others), whatever the deletion order.

Result 2: the leader rejects every link

Outside the sign condition, at $\lambda = 1$:

- The leader gains from its own innovation and loses from rivals'.
- Every incident link passes at least a fraction ϕ_* of its extra output to other groups.



Leader isolated; followers can link (Corollary 2).

Proposition 2

If an admissible exposure satisfies $\eta \geq \dfrac{Da_1}{\Delta\,\phi_*}$, the leader rejects every incident link, in every network.

The leader values its own innovation but rejects links that strengthen rivals.

$\phi_* = [1-\beta(N-1)]^2/(2R_v)$. $v_k = \mathbb{E}[\xi_k]$: mean innovation input of group k. $R_v = \max_k v_k / \min_k v_k$.

To do

1. **Existence:** general mixed-sign valuations, where equilibrium links can be interior.

2. **More general ranking probabilities:** nonlinear rules, such as Plackett–Luce.

3. **Dynamics:**
   - links are formed every period; today's ranking is tomorrow's starting point;
   - subgame perfect equilibrium;
   - does the hierarchy persist, or do innovators take over?