Pontificia Universidad Católica del Perú
Economics Major

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Recitation 6
ECO 263

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Exercise 1. Let us consider a scenario where three individuals living in a building are considering the construction of an elevator, which costs $C = 2100$ units of money. The individuals have preferences that depend on the money m^i[1], and on x, which takes the value of 1 if there is an elevator and 0 if there is not. The initial wealth that each individual has is w^i for $i = 1, 2, 3$.

(a) Suppose that preferences can be represented by the following Stone-Geary type utility functions:

$$U^i(m^i, x) = \sqrt{m^i}(x+i)^{0.5},\ i = 1, 2, 3.$$

Find the reservation price of each individual. Then, assuming an endowment of 5000 soles, analyze if the public good should be provided. In the general case, show that this will depend on the wealth distribution $\{w_1, w_2, w_3\}$.

(b) Now suppose that preferences can be represented by quasilinear utility functions:

$$U^i(m^i, x) = m^i + 300(i+1)(x)^{0.5},\ i = 1, 2, 3.$$

Find the reservation price of each individual. Assess whether the provision of the public good is efficient or not, and show that, unlike the previous case, your answer does not depend on the distribution of wealth.

(c) What is the individual contribution that maximizes the welfare of agent i given the contributions made by the rest of the community members?

(d) Assuming the costs are shared equally, calculate the net value v_i of the elevator installation for each individual. If majority voting is used as the decision mechanism for the provision of the public good, who votes in favor and who votes against? Is the public good installed?

[1] Which is used to consume other goods.

Exercise 2. Consider a scenario where three individuals are evaluating the provision of a public good with a cost of $C = 330$ monetary units. The individuals have preferences that depend on the amount of money m^i they have to consume other goods and on G (the public good, which takes the value 0 if it is not provided and 1 otherwise):

$$U^i(m^i, G) = m^i + 50(2^{i-1})\sqrt{G}, \quad i = 1, 2, 3.$$

The wealth of each individual is denoted by w^i, $i = 1, 2, 3$.

a) Find the reservation price of each individual. Evaluate whether the provision of the public good is efficient or not.

b) Suppose that if the public good is provided, the cost is shared equally, $s_i = 1/3$. Find the corresponding net value for each individual.

c) If majority voting is used as the decision mechanism for the provision of the public good, who votes in favor and who votes against? Is the public good provided?

d) Suppose that each agent contributes payments in proportion to how much they value the public good, $s_i = \frac{r_i}{\sum r_i}$. Who votes in favor and who votes against? Is the public good provided? What is the problem with this mechanism?

e) Suppose the financing of the public good is based on equal payments. Assume the Groves-Clarke mechanism is applied, such that the public good is provided if the sum of the net reported values of each individual is greater than zero ($\sum_i \tilde{v}_i \geq 0$), and if the good is provided, side payments are given to each individual equal to the sum of the reported valuations of the others ($\sum_{j \neq i} \tilde{v}_j$). Express the profit function of each individual.

Continuous Public Goods

Exercise 3. Let G be a public good and x a private good. The utility of the individuals can be expressed as follows:

$$u^h(G, x^h) = \alpha \ln G + \beta \ln x^h, \; h = 1, 2, \; \alpha, \beta \in (0, 1).$$

The production function for the public good is expressed as:

$$G = f(z) = \theta z, \quad \theta > 0.$$

Finally, the endowments are w^h, $h = 1, 2$.

a) Find the Pareto optimal level of provision for the public good G.

b) Determine the optimal level of production for the public good. Assume that the price of the good is equal to 1.

c) Will the provision of the public good be efficient in competitive equilibrium? Justify your answer.

d) Does the provision of the public good change when a Lindahl tax is introduced? How does this result compare to what is described in part (c)?

Optional problems

Exercise 4. Consider two consumers $i = 1, 2$, each one with an income $w_1 = w_2 = w > 0$ to allocate between two goods. Good 1 provides a unit of consumption to its purchase and $\alpha \in (0, 1)$ units of consumption to the other consumer. Each consumer has a quasilinear utility function $u_i = \ln x_1^i + x_2^i$.

a) Provide an interpretation of the parameter α.

b) Assume that good 2 is a private good. Find the optimal level of consumption assuming that the price of both goods is equal to one.

c) By maximizing the sum of utilities, show that the equilibrium is Pareto efficient if $\alpha = 0$ but inefficient for all other values of α.

d) Assume that good 2 provides 1 unit of consumption to its purchaser and $\alpha \in (0, 1)$ to the other consumer. Obtain the Nash equilibrium and show that it is efficient for all values of α.

Exercise 5. **From Mas-Colell, Whinston and Green (11.D.7).** A continuum of individuals can build their houses in one of two neighborhoods, A or B. It costs c_A to build a house in neighborhood A and $c_B < c_A$ to build in neighborhood B. Individuals care about the prestige of the people living in their neighborhood. Each individual has a level of prestige, denoted by the parameter θ, where θ is uniformly distributed across the population on the interval $[0, 1]$.

The prestige of neighborhood k (where $k = A, B$) is represented by the average prestige value of individuals in that neighborhood, denoted by θ_k. If an individual i with prestige parameter θ chooses to build their house in neighborhood k, their derived utility, net of building costs, is given by

$$(1 + \theta)(1 + \theta_k) - c_k.$$

Thus, individuals with higher prestige value living in a prestigious neighborhood more. Assume that c_A and c_B are both less than 1 and that the cost difference $(c_A - c_B)$ falls within the interval $\left(\frac{1}{2}, 1\right)$.

(a) Show that in any building-choice equilibrium (technically, the Nash equilibrium of the simultaneous-move game where individuals simultaneously choose where to build their house), both neighborhoods must be occupied.

(b) Show that in any equilibrium where the prestige levels of the two neighborhoods differ, every resident of neighborhood A must have at least as high a prestige level as every resident of neighborhood B. This implies the existence of a cutoff prestige level $\hat{\theta}$ such that all types $\theta \geq \hat{\theta}$ build in neighborhood A, and all $\theta < \hat{\theta}$ build in neighborhood B. Characterize this cutoff level.

(c) Show that in any equilibrium of the type identified in part (b), a Pareto improvement can be achieved by adjusting the cutoff value θ slightly and allowing transfers between individuals.