FACULTY OF SOCIAL SCIENCES
ECONOMICS MAJOR

Recitation 5

Microeconomics 2
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1 Monopoly

Exercise 1.1. An economy has two types of consumers and two goods. The agent type A has the following utility function:

$$u_A(x_{1A}, x_{2A}) = 4x_{1A} - \frac{x_{1A}^2}{3} + x_{2A}$$

and the agent type B has the following utility function:

$$u_B(x_{1B}, x_{2B}) = 3x_{1B} - \frac{x_{1B}^2}{2} + x_{2B}.$$

Good 2 is the numeraire, and each consumer has an income of 100. Additionally, the economy has N consumers of both type A and type B.

1. Identify the type of consumer with high demand and the type with low demand for good x_1. Compare the marginal willingness to pay for each type of consumer for good x_1.

2. The monopolist produces good 1 with the following cost function $C(x_1) = cx_1$ and cannot discriminate prices. Find the optimal price and quantity of good x_1 that the monopolist will choose. For which values of c will the monopolist choose to sell to both types of consumers?

3. The monopolist engages in second-degree price discrimination by offering a menu of prices and quantities to each type of consumer (r_A, x_A) and (r_B, x_B). Based on this, formulate the monopolist's optimization problem and find the optimal values (r_A^*, x_A^*) and (r_B^*, x_B^*).

4. If the monopolist engages in third-degree price discrimination, what will be the prices and quantities set by the monopolist in the markets for A-type and B-type consumers?

5. If the monopolist engages in first-degree price discrimination, find the quantity produced by the monopolist in the market for good x. Calculate the consumer surplus and the monopolist's surplus.

Exercise 1.2. From Tirole (1994). Consider $q(p) = p^{-\epsilon}$ and assume constant marginal cost. Prove that the social welfare in competitive equilibrium is

$$\mathcal{W}^s = \frac{c^{1-\epsilon}}{\epsilon - 1}.$$

Then, compute the welfare loss. *Hint*: recall that the total surplus is in the competitive case

$$\int_{p=c'}^{\infty} q(s)ds.$$

So, in this case, $\int_{p=c}^{\infty} s^{-\epsilon}ds = \frac{c^{1-\epsilon}}{\epsilon-1}$. For the monopolist case, apply FOC

$$p^m = \frac{c}{1 - \frac{1}{\epsilon}}.$$

Thus, you can conclude that

$$\mathcal{W}^s - \mathcal{W}^m = \left(\frac{c^{1-\epsilon}}{\epsilon - 1}\right)\left[1 - \left(\frac{2\epsilon - 1}{\epsilon - 1}\right)\left(\frac{\epsilon}{\epsilon - 1}\right)^{-\epsilon}\right].$$

2 Externalities

Exercise 2.1. Consider an economy where there are two goods (y_1, y_2), two consumers, and a firm. Each consumer has an initial endowment of four units of good 1 and nothing of good 2. Good y_1 is not producible, and good y_2 is produced by the firm using good 1 as input (the quantities of good 1 not directly consumed by individuals) based on the following production function:

$$y_2 = f(z) = z_1^{1/2},$$

where y_2 is the quantity of good 2 produced and z_1 is the amount of good 1 used as input. The profits earned by the firm are equally distributed between the two consumers, $\theta_j = 1/2$. Both consumers derive utility from the consumption of the two goods. However, the production of good 2 generates noise and pollution, which negatively affects their well-being. As a result, the utility function of the consumers is given by the following expression:

$$u_i(y_{i1}, y_{i2}, y_2) = y_{i1} + \ln y_{i2} - \frac{1}{2}\ln y_2, \quad i = 1, 2.$$

The superscript refers to the consumer.

a) Calculate the quantities of good 2 produced and consumed by the two individuals in the general equilibrium, assuming the price of good 1 as the numéraire, whether used as a consumption good or as an input ($p_1 = w_1 = 1$).

b) Calculate the quantities of good 2 produced and consumed by the two individuals in the efficient allocation, and comment on the results in comparison to the previous part.

Exercise 2.2. The company S produces a certain amount of steel (s) and a certain amount of pollution (x), which is discharged into a river. The company F is a fish farm located downstream and is negatively affected by the pollution from company S. Suppose that the cost function of S involves both s and x. Meanwhile, the company F depends on f, representing the collection of fish, and x, which represents the production of pollution. Additionally, it must be considered that pollution increases the cost of fish production and reduces the cost of steel production.

a) Formulate the profit maximization problem for both companies.

b) What are the conditions that characterize profit maximization? Remember that polluting has no price.

c) How would the efficient production plan of steel and fish in the Pareto sense look? What are the implications of this new scenario for pollution production?

Lima, October 7, 2024.

References

Tirole, J. (1994). *The Theory of Industrial Organization.* MIT University Press.