FACULTY OF SOCIAL SCIENCES
ECONOMICS MAJOR

Recitation 3

Microeconomics 2
Semester 2024-2

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1 Efficiency and Welfare Theorems

Exercise 1.1. State the First Welfare Theorem and prove it. See Echenique (2015).

Exercise 1.2. State the Second Welfare Theorem. Any idea of how to prove it? See Echenique (2015) or Varian (1992).

Exercise 1.3. Imagine an exchange economy composed of two individuals A and B. The preferences of these individuals are represented by the following utility functions:

$$u_A(x_A, y_A) = x_A y_A^{1/2}$$
$$u_B(x_B, y_B) = x_B^{1/2} y_B.$$

The endowments are $\omega_1 = (100, 0)$ and $\omega_2 = (0, 150)$.

- Find and characterize the Pareto set or contract curve of this economy.
- Calculate the Walrasian equilibrium of this economy given the initial endowments stated in the problem. Show that the allocation found belongs to the Pareto set. Relate this to the First Welfare Theorem.
- Choose any other point on the Pareto set, and indicate a way to reach it through competitive equilibrium, proposing transfers between the individuals to make it possible. Relate this to the Second Welfare Theorem.

2 Robinson-Crusoe Economy

Exercise 2.1. Consider a Robinson Crusoe economy where

$$u(\ell_o, c) = \ell_o^2 c$$
$$f(\ell_t) = \sqrt{\ell_t},\ \bar{\ell} = 10.$$

ℓ_t denotes the hours worked and ℓ_0 the hours of leisure.

1. Solve the problem in a centralized manner.

2. Solve the problem from the market approach.

3. **Additional exercise:** Now consider $u(\ell_0, c) = \ell_0^{1/2} + c^{1/2}$ and $f(\ell_t) = \sqrt{\ell_t}$. Again, take $\bar{\ell} = 10$. Moreover, take $f(\ell_t) = \ell_t$ and then $f(\ell_t) = \ell_t^2$. What changes?

Exercise 2.2. Suppose Acemoglu is a castaway living on an island and can gather apples with a technology given by $y = 4\ell_t^{1/2}$. His utility function is $u(y, \ell_o) = 2y\ell_o$, where ℓ_t is the number of hours worked and ℓ_o is the number of hours of leisure. We know that he has 24 hours available each day to either gather apples or rest.

a) Formulate the consumer problem specifying the control (optimization) variables.

b) Find Acemoglu's optimal consumption (apples and leisure). What is the profit that Acemoglu obtains as a producer and what is the level of welfare he reaches as a consumer?

c) What is the shadow relative price of both goods?

3 Economies with Production

Exercise 3.1. Consider an economy with two goods, two consumers (Obi-Wan and Palpatine), and a firm (Sereno). Obi-Wan has preferences represented by $u_1(x_1, y_1) = \sqrt{x_1 y_1}$, with an initial endowment of $\boldsymbol{\omega}_1 = (1, 0)$ and $\theta_1 = 0.3$. Palpatine has quasilinear preferences $u_2(x_2, y_2) = x_2 + \ln(y_2)$, with an initial endowment of $\boldsymbol{\omega}_2 = (2, 0)$ and $\theta_2 = 0.7$. On the other hand, the firm's technology is

$$Y = \left\{ (x, y) \in \mathbb{R}^2 : x \leq 0, y \leq \frac{Ax}{x-1} \right\}$$

where $A > 0$ is a productivity factor.

1. Find Sereno's supply function.

2. Find Obi-Wan's and Palpatine's demand correspondence.

3. Find the Walrasian equilibrium.

4. Study the effect of the productivity factor A on the equilibrium price ratio.

Exercise 3.2. [Graded Recitation 2024-1]. Consider an economy comprised of two agents named Keynes and Solow who derive utility from the consumption of a good and leisure (x_i, h_i). The good (x) us produced by using only one productivity factor, labour. The production function that reflects the technological characteristics of the labour transformation process is given by $x = f(L) = 4L^{0.5}$. Keynes has preferences represented by $u_1(x_1, h_1) = x_1 + \ln(h_1)$. Solow, on the other hand, has preferences represented by $u_2(x_2, y_2) = 2x_2 + 2\ln(h_2)$. Suppose that the agents in the economy have decided to use competitive market institutions to organize the production and distribution of goods in the economy. Under this mechanism, each agent has the following initial endowment $w_i = (x_i = 0, \ell_i + h_i = 10)$. Additionally, only agent 1 (Keynes) receives 100% of the firm's profits.

1. Formulate and solve the firm's optimization problem. Find the quantity of labour demanded, the production, and the firm's profits.

2. Formulate and solve the consumers' optimization problem. Find the individual demands and the respective supply functions.

3. Find the Walrasian equilibrium (prices, consumption bundles, and quantities of the production factor used).

4 Advanced Exercises

Exercise 4.1. Prove that, in a pure exchange economy, an allocation

$$\mathbf{x}^* = (\mathbf{x}_1^*, \mathbf{x}_2^*, \cdots, \mathbf{x}_I^*) \in \mathbb{R}_+^{IL}$$

where

$$\mathbf{x}_i = (x_{1i}^*, \cdots, x_{Li}^*) \in \mathbb{R}_+^{L}$$

is a Pareto optimum if and only if $\mathbf{x}^*$ solves the following optimization problem

$$\begin{aligned} &\max\ u_1(\mathbf{x}_1) \\ &\text{s. t. } u_i(\mathbf{x}_i) \geq \overline{u}_i,\ i \neq 1 \\ &\qquad \sum_{i=1}^{I} \mathbf{x}_i \leq \sum_{i=1}^{I} \boldsymbol{\omega}_i \\ &\qquad \mathbf{x}_i \geq \mathbf{0},\ \forall\ i = 1, ..., I. \end{aligned}$$

where $\overline{u} > 0$ is an arbitrary utility level. Assume that preferences are strongly monotone, continuous, and such that $u_i(\mathbf{0}) = 0$ for $i = 1, 2, ..., I$.

Exercise 4.2. Prove that, in a pure exchange economy, if preferences are locally non satiated, then every Walrasian equilibrium is Pareto optimal (note that this is the 1st Welfare Theorem). *Recall that, $\succeq$ is locally non satiated over $X = \mathbb{R}^L$ if for every $\boldsymbol{x} \in X$ and $\epsilon > 0$, there exists $\boldsymbol{y} \in \mathcal{B}(\boldsymbol{x}, \epsilon) = \{\boldsymbol{z} \in \mathbb{R}^L : ||\boldsymbol{x} - \boldsymbol{z}|| = \sqrt{\sum_{i=1}^{L}(x_i - z_i)^2} < \epsilon\}$ such that $\boldsymbol{y} \succ \boldsymbol{x}$.*

Exercise 4.3. Prove that in a pure exchange economy where preferences are continuous, strongly monotonic and strongly convex there exists $\mathbf{p}^*$ such that

$$z(\mathbf{p}^*) = \sum_{i=1}^{I} x_i(\mathbf{p}^*) - \sum_{i=1}^{I} \boldsymbol{\omega}_i \leq \mathbf{0}.$$

Hint: consider $g(\mathbf{p}) = \left[\dfrac{p_\ell + \max\{0, z_\ell\}}{1 + \sum_{\ell=1}^{L} \max\{0, z_\ell\}} \right]_{\ell=1,\ldots,L}$ and Brouwer fixed point theorem.

Exercise 4.4 (Midterm 2024-1)**.** . Consider a 2×2 economy where

$$u_1(x_{11}, x_{21}) = x_{11} + 2x_{21},\ \boldsymbol{\omega}_1 = (1, 0)$$
$$u_2(x_{12}, x_{22}) = 2x_{12} + x_{22},\ \boldsymbol{\omega}_2 = (0, 1).$$

1. Derive the optimal demands for each consumer in terms of the price ratio p_1/p_2.

2. Obtain a Walrasian equilibrium. Explain why it is also a Pareto optimal allocation. Finally, obtain all Pareto Optimal allocations.

Exercise 4.5. Consider a 2×2 economy where

$$u_1(x_{11}, x_{21}) = \max\{x_{11}, x_{21}\}$$
$$u_2(x_{12}, x_{22}) = \min\{x_{12}, x_{22}\}$$

and $\boldsymbol{\omega}_1 = (1, 0)$, $\boldsymbol{\omega}_2 = (0, 1)$.

1. Derive the optimal demands for each consumer in terms of the price ratio p_1/p_2.

2. If possible, obtain a Walrasian equilibrium. Explain why it is also a Pareto optimal allocation.

3. Obtain all Pareto Optimal allocations.

Exercise 4.6. Prove that in a 2×2 economy where

$$u_1(x, y) = \max\{\min\{x, y/2\}, \min\{x/2, y\}\}$$
$$u_2(x, y) = \min\{x, y\}.$$

and $\boldsymbol{\omega}_1 = \boldsymbol{\omega}_2 = (1, 1)$, there is no Walrasian equilibrium. Why?

5 More advanced exercises in production theroy

Adapted from Mas-Colell et al. (1995), Chapter 5. The definition of convex cone is in Ok (2007).

Exercise 5.1. Let $Y \subset \mathbb{R}^n$ be a technology. We will say that the technology exhibits non-increasing returns to scale if: $\forall\ \mathbf{y} \in Y$, $\alpha\mathbf{y} \in Y$, $\forall\ \alpha \in [0, 1]$. On the other hand, we will say that the technology is additive if given $\mathbf{y}, \mathbf{y}' \in Y$, $\mathbf{y} + \mathbf{y}' \in Y$. Prove that a technology exhibits non-increasing returns to scale and is additive if and only if it is a convex cone.

Exercise 5.2. It is said that a technology $Y \subset \mathbb{R}^L$ has the property of free disposal if given $\mathbf{y} \in Y$ and $\mathbf{y}' \leq \mathbf{y}$, then $\mathbf{y}' \in Y$. Prove that if a technology is closed (i.e., Y is a closed set), convex, and such that $-\mathbb{R}^L_+ \subset Y$, then it satisfies the property of free disposal.

Lima, September 16, 2024.

References

Echenique, F. (2015). Lecture notes general equilibrium theory.

Mas-Colell, A., Whinston, M. D., and Green, J. R. (1995). *Microeconomic Theory*. Oxford University Press, New York.

Ok, E. A. (2007). *Real Analysis with Economic Applications*. Princeton University Press, Princeton, NJ.

Varian, H. R. (1992). *Microeconomic Analysis*. W. W. Norton & Company, New York, 3rd edition.