

## Recitation 2

Microeconomics 2  
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### 1 $2 \times 2$ Economy

**Exercise 1.1.** In each of the following cases, draw the Edgeworth box, some indifference curves for each consumer and find Walrasian (competitive) equilibrium in each case. **Later on, you should be able to find the Pareto set and the core (contract curve).**

- a)  $u_1(x_{11}, x_{21}) = 2x_{11}^2 x_{21}$ ,  $u_2(x_{12}, x_{22}) = x_{12} x_{22}^3$ ,  $\omega_1 = (2, 3)$  and  $\omega_2 = (1, 2)$ .
- b)  $u_1(x_{11}, x_{21}) = x_{11} + x_{21}$ ,  $u_2(x_{12}, x_{22}) = \min\{x_{12}, x_{22}\}$ ,  $\omega_1 = (1, 2)$  and  $\omega_2 = (3, 4)$ .
- c)  $u_1(x_{11}, x_{21}) = x_{11} + \ln x_{21}$ ,  $u_2(x_{12}, x_{22}) = x_{12} + 2 \ln x_{22}$ ,  $\omega_1 = (2, 3)$  and  $\omega_2 = (1, 2)$ .
- d)  $u_1(x_{11}, x_{21}) = x_{11} x_{21}$ ,  $u_2(x_{12}, x_{22}) = \min\{x_{12}, x_{22}\}$ ,  $\omega_1 = (2, 6)$  and  $\omega_2 = (4, 1)$ .
- e)  $u_1(x_{11}, x_{21}) = \min\{2x_{11}, x_{21}\}$ ,  $u_2(x_{12}, x_{22}) = \min\{x_{12}, 2x_{22}\}$ ,  $\omega_1 = (1, 2)$  and  $\omega_2 = (3, 4)$ .
- f)  $u_1(x_{11}, x_{21}) = 3x_{11} + x_{21}$ ,  $u_2(x_{12}, x_{22}) = x_{12} + 3x_{22}$ ,  $\omega_1 = (2, 2)$  and  $\omega_2 = (2, 2)$ .

Identify whenever it is possible the type (Cobb-Douglas, CES, Leontief, linear...) of the utility function.

**Exercise 1.2.** From [Mas-Colell et al. \(1995\)](#). Consider a  $2 \times 2$  economy in which consumers preferences are monotonic. Prove that (here below  $\omega_\ell = \omega_{1\ell} + \omega_{2\ell}$ )

$$p_1 \left( \sum_{i=1}^2 x_{1i}(p_1, p_2) - \omega_1 \right) + p_2 \left( \sum_{i=1}^2 x_{2i}(p_1, p_2) - \omega_2 \right) = 0.$$

Use this to explain Walras law, *if one market clears the other too*. Generalize this result to  $I$  consumers and  $L$  goods.

**Exercise 1.3.** From [Mas-Colell et al. \(1995\)](#). Consider an Edgeworth box economy in which each consumer has Cobb-Douglas preferences

$$\begin{aligned} u_1(x_{11}, x_{21}) &= x_{11}^\alpha x_{21}^{1-\alpha} \\ u_2(x_{12}, x_{22}) &= x_{12}^\beta x_{22}^{1-\beta}, \end{aligned}$$

with  $\alpha, \beta \in (0, 1)$ . Consider endowments  $(\omega_{1i}, \omega_{2i}) > 0$  for  $i = 1, 2$ . Solve for the equilibrium price ratio and allocation.

**Exercise 1.4.** There are two consumers,  $A$  and  $B$ , with the following utility functions,

$$\begin{aligned} u_A(x_A^1, x_A^2) &= a \ln x_A^1 + (1-a) \ln x_A^2, \quad \omega_1 = (0, 1) \\ u_B(x_B^1, x_B^2) &= \min\{x_B^1, x_B^2\}, \quad \omega_2 = (1, 0). \end{aligned}$$

Compute the prices and quantities that clear the market. Interpret.

**Exercise 1.5.** Consider two individuals in a pure exchange ( $2 \times 2$ ) economy whose indirect utilities are

$$\begin{aligned} v_1(p_1, p_2, w) &= \frac{w}{p_1 + p_2} \\ v_2(p_1, p_2, w) &= \frac{abw}{bp_1 + ap_2}, \quad a, b > 0. \end{aligned}$$

Endowments are  $\omega_1 = (1, 1)$  and  $\omega_2 = (1, 1)$ . Obtain the equation that prices which clear the market must satisfy. *Hint:* apply Roy's identity.

## 2 Additional exercises

**Exercise 2.1.** Suppose that in a  $2 \times 2$  economy consumer  $i$  has Cobb-Douglas preferences  $u_i(x_{1i}, x_{2i}) = x_{1i}^\alpha x_{2i}^{1-\alpha}$ . Furthermore, assume that endowments are  $\omega_1 = (1, 2)$  and  $\omega_2 = (2, 1)$ . Find the (a)<sup>1</sup> Walrasian equilibrium. **Later on, you should be able to find the optimal Pareto assignments.**

**Exercise 2.2. For when you've seen Pareto Optimality in class.** Under some conditions over the preferences, in a  $2 \times 2$  economy, every Pareto Optimal allocation can be characterized as the solution of the following maximization problem (you should try to prove it)

$$\begin{aligned} \max \quad & u_1(\mathbf{x}_1) \\ \text{s. t.} \quad & u_2(\mathbf{x}_2) \geq k \\ & \mathbf{x}_1 + \mathbf{x}_2 = \boldsymbol{\omega}_1 + \boldsymbol{\omega}_2, \end{aligned}$$

where  $k \in \mathbb{R}$ . Find the aforementioned conditions over the preferences. Generalize this result for pure exchange economies with  $I$  individual and  $L$  goods.

**Exercise 2.3. Medium-difficulty.** From [Chavez and Gallardo \(2024\)](#). Consider an

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<sup>1</sup>We don't know if it is unique or no! However, under some conditions over the preferences, which are satisfied in this exercise, existence is ensured.

economy with  $N$  consumers, two goods, and preferences given by

$$u_i(x_{1i}, x_{2i}) = x_{1i}^2 + x_{2i}^2.$$

Endowments are  $\omega_i = (1, 1)$ . If  $N$  is even, find, if it exists, a Walrasian equilibrium. What if  $N$  is odd?

**Exercise 2.4. Mandatory to know.** Prove that if  $\succeq$  is monotone, then it is locally non satiated. Here  $\succeq$  represents a preference relation over  $\mathbb{R}_+^L$ .

**Exercise 2.5. Medium difficulty. For when you've seen Pareto Optimality in class.** Prove 1st Welfare theorem for a  $2 \times 2$  economy. This is, if preferences are locally non satiated, then, every Walrasian equilibrium is Pareto optimal. Can you generalize this for a pure exchange economy with  $N$  consumers and  $L$  goods? You can guide yourself from [Echenique \(2015\)](#).

Lima, September 2, 2024.

## References

- Chavez, J. and Gallardo, M. (2024). *Algebra Lineal y Optimization para el Análisis Económico*. Prepublished.
- Echenique, F. (2015). Lecture notes general equilibrium theory.
- Mas-Colell, A., Whinston, M. D., and Green, J. R. (1995). *Microeconomic Theory*. Oxford University Press, New York.