

Second Test

Microeconomics 2
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- You have 115 minutes.
- Be clear in your solutions. Justify each step.
- Up to 22 points.
- Closed books and all electronic devices are forbidden.
- You may assume any result covered in class.

Exercise 1 (4 points). For items (1) and (2), analyze if the statement is true or not and **justify**. For (1), (3) and (4), give the Pareto optimal allocations as a detailed draw.

1. Consider the following utility functions $u_1(x_1, y_1) = x_1 + y_1$ and $u_2(x_2, y_2) = \max\{x_2, y_2\}$. Is $\{(0, 0), (2, 2)\}$ a Pareto optimal allocation? Is it a weak Pareto optimal allocation?
2. Given a 2×2 economy where consumers have the following utility functions: $u_i(x_{1i}, x_{2i}) = \sqrt{x_{1i}^2 + x_{2i}^2}$, then the Pareto set does not exist.
3. Obtain the Pareto set for a 2×2 economy where preferences are given by

$$u_i(x_i, y_i) = \sqrt{x_i} + \sqrt{y_i}.$$

4. Endowments are $\omega_1 = (2, 3)$ and $\omega_2 = (3, 2)$. Preferences are represented by $u_i(x_i, y_i) = 2 \ln x_i + \ln y_i$. Obtain the contract curve.

Exercise 2 (4 points). Consider a Robinson Crusoe economy where

$$u(\ell_o, c) = \ell_o c$$

$$f(\ell_t) = \ell_t$$

$$\bar{\ell} = 24.$$

Remember that $\ell_t + \ell_o = \bar{\ell}$.

1. Solve the problem in a centralized manner. This involves directly substituting the constraints into the optimization problem, all in terms of ℓ_t .
2. Solve the problem from a market perspective. **Hint:** argue why the equilibrium only exists if $p = w$.

Exercise 3 (6 points). Consider an economy called Sommerville, consisting of two consumers (Carlos and Brik), two goods, and a firm. The agents consume two goods: papers (x) and books (y). However, the agents only have initial endowments of papers, $\omega_1 = (4, 0)$ and $\omega_2 = (5, 0)$ respectively. On the other hand, the only firm produces books with the following technology

$$Y = \{(x, y) \in \mathbb{R}^2 | x \leq 0, y \leq 2\sqrt{-x}\}.$$

Moreover, the preferences of the consumers are represented by $u_1(x_1, y_1) = x_1^{1/2} y_1^{1/2}$ and $u_2(x_2, y_2) = x_2^{1/4} y_2^{3/4}$, respectively. Shares are $\theta = (\theta_1, \theta_2) = (0.1, 0.9)$.

- a) Find the firm's input demand function for papers (x^d), the firm's supply function (y^s), and the profits π^* . Seek only for interior solutions.
- b) Find the demands for goods x and y for each consumer.
- c) Find the Walrasian equilibrium, this is, the quantities consumed by each agent for each good, the input quantity used (x), and the firm's production (y).

Exercise 4 (4 points). In the Economics Department at PUCP, the only seller of algorithms (x), Manuel, faces an inverse demand curve given by $p = a - bq$, where $a, b > 0$ and p is the price per algorithm sold. We assume that an algorithm is a perfectly divisible good, so $q \in \mathbb{R}_+$. Manuel has a quadratic cost function $C(q) = \gamma q^2 + \bar{c}$ in the number of algorithms sold ($\gamma, \bar{c} > 0$ are parameters).

1. Find the quantity of algorithms that Manuel sells q^m and the price at which he sells them p^m . Remember that Manuel, given the context, operates as a monopolist. Your answer will depend on a and b . For your answer to make sense ($q^m \geq 0$), which is the relation that a and b must satisfy?
2. What happens with q^m and p^m if γ increases?
3. If the fixed cost changes to $2\bar{c}$, do any of the previous answers changes? Why?

Exercise 5 (2 points). Many exporting firms sell in two markets, the domestic market and the global market. They are typically price takers in the global market but also tend to have market power in the domestic market. The demand in the domestic market is given by:

$$y^d = 100 - p^d.$$

On the other hand, their costs are given by $C(y^d, y) = 0.5(y^d + y)^2$ and the exchange rate is equal to 3, and $p = 20USD$ (in the international market). Obtain the optimal y^d , y^* , p^d , and the monopolist's profits.

Viernes económicos (2 points)

- a) En el 2014, el 25% de la deforestación en el Perú estuvo asociada al cultivo de _____.
- b) Los factores ESG son _____, como el cambio climático; _____, que incluyen temas como derechos humanos; y de _____, como es el caso de la transparencia.