FACULTY OF SOCIAL SCIENCES
ECONOMICS MAJOR

Solutions to the Second Test

Microeconomics 2
Semester 2024-2
September 30

Professor: Pavel Coronado Castellanos
pavel.coronado@pucp.edu.pe
Teaching Assistants: Marcelo Gallardo & Fernanda Crousillat
marcelo.gallardo@pucp.edu.pe
https://marcelogallardob.github.io/
a20216775@pucp.edu.pe

Exercise 1 (4 points). For items (1) and (2), analyze if the statement is true or not and **justify**. For (1), (3) and (4), give the Pareto optimal allocations as a detailed draw.

1. In a 2×2 economy, if preferences are represented by $u_i(x_{1i}, x_{2i}) = \exp(x_{1i}^2 + x_{2i}^2)$, then the Pareto set does not exist.

2. Alice and Bob's utilities are

$$U^A(x_1^A, x_2^A) = x_1^A, \quad U^B(x_1^B, x_2^B) = x_2^B.$$

   Then, $\mathbf{x} = \{(3,3), (0,0)\}$ is a Pareto optimal allocation.

3. Alice and Bobs' utilities are $u_A(x^A, y^A) = \ln x^A + y^A$ and $u_B(x^B, y^B) = x^B$. Find Pareto optimal allocations.

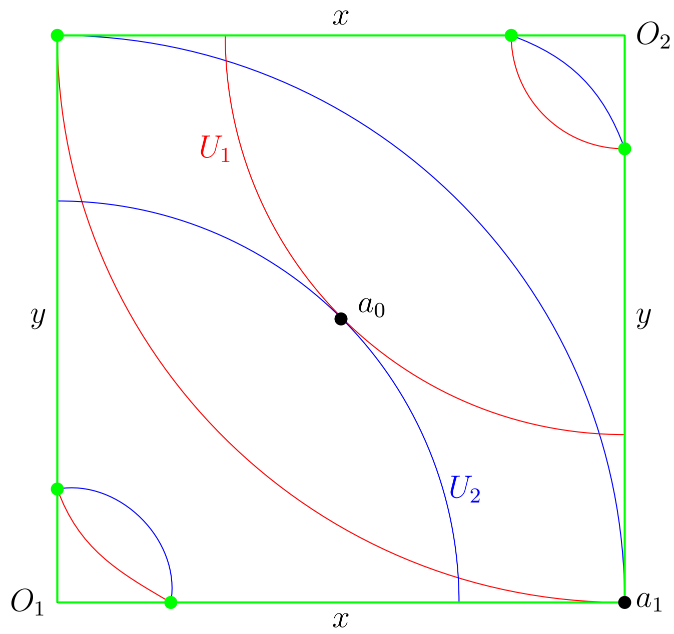
4. Obtain the contract curve for a 2×2 economy in which

$$\underbrace{u_i(x_i, y_i) = (x_i - 1)^{\beta_1}(y_i - 1)^{\beta_2}}_{\text{Stone-Geary utility function}}, \ \beta_i > 0$$

   and $\boldsymbol{\omega}_1 = (3,2)$, $\boldsymbol{\omega}_2 = (2,3)$.

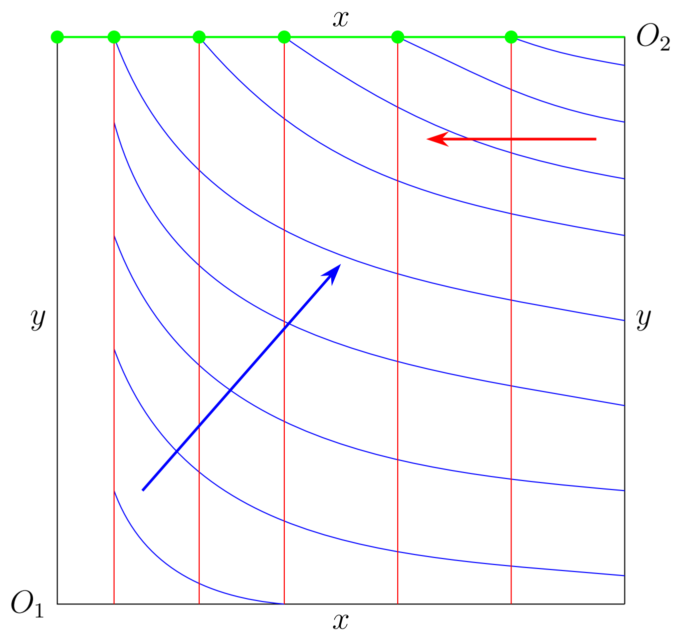
Solution:

1.a) False: the Pareto set is the frontier of the Edgeworth box. See the following figure.

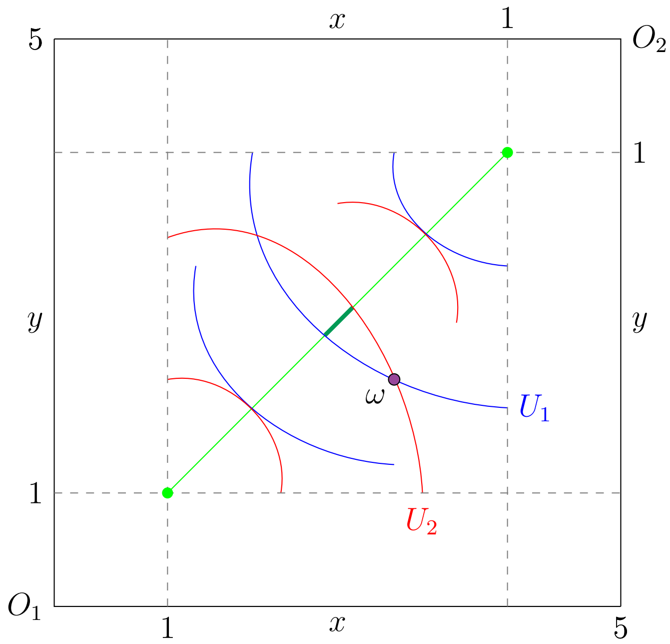


1.b) False, $\{(3,0),(0,3)\}$ strictly dominates the initial allocation.

1.c) The Pareto set (drawing the indifference curves) is the upper side of the Edgeworth box. See the following figure.



1.d) Applying $MRS_1 = MRS_2$, and $x_1 + x_2 = 5 = y_1 + y_2$, we easily compute $\mathcal{P} = \{(x_1, y_1) \in [0,5]^2 : \ y_1 = x_1\} \cap S$ where S is determined by the endowments. See the following figure.



Exercise 2 (**4 points.**). Consider a Robinson Crusoe economy where

$$u(\ell_o, c) = \sqrt{\ell_o c}$$
$$f(\ell_t) = \sqrt{\ell_t}$$
$$\bar{\ell} = 24.$$

Remember that $\ell_t + \ell_o = \bar{\ell}$.

1. Solve the problem in a centralized manner. This involves directly substituting the constraints into the optimization problem, all in terms of ℓ_t. Be clear why the solution is or (is not) interior.

2. Solve the problem from a market perspective.

Solution:
2.1) The centralized problem is

$$\max_{0 \leq \ell_t \leq \bar{L}} \ \ell_t^{1/4}(24 - \ell_t)^{1/2}.$$

First order condition leads to $\ell_t = 8$, $\ell_o = 16$ and $c = \sqrt{8}$.

2.2) Via the market, profits maximization leads to the following problem:

$$\max_{\ell_t \in [0,24]} \ p\sqrt{\ell_t} - w\ell_t$$

Again, by Inada condition the solution is interior an FOC applied. We obtain $\ell_t^d = \frac{p^2}{4w^2}$. Hence, $\Pi = \frac{p^2}{4w}$ and $c^s = \frac{p}{2w}$. On the other hand, optimal demands (since we are working with Cobb-Douglas) are $\ell_o^d = \frac{1}{2w}(\Pi + 24w)$ and $c^d = \frac{1}{2p}(\Pi + 24w)$. Clearing the first markets leads to $p/w = 4\sqrt{2}$ and, as in the first item, $\ell_t = 8$, $\ell_o = 16$ and $c = \sqrt{8}$.

Exercise 3 (6 points). Consider an economy called Sommerville, consisting of two consumers (Carlos and Brik), two goods, and a firm. The agents consume two goods: papers (x) and books (y). However, the agents only have initial endowments of papers, $\boldsymbol{\omega}_1 = (3, 0)$ and $\boldsymbol{\omega}_2 = (2, 0)$ respectively. On the other hand, the only firm produces books with the following technology

$$Y = \{(x, y) \in \mathbb{R}^2 | x \leq 0,\ y \leq \sqrt{-x}\}.$$

Moreover, the preferences of the consumers are represented by $u_1(x_1, y_1) = \sqrt{x_1 y_1}$ and $u_2(x_2, y_2) = 2 \ln x_2 + \ln y_2$, respectively. Shares are $\boldsymbol{\theta} = (\theta_1, \theta_2) = (0.5, 0.5)$.

a) Find the firm's input demand function for papers (x^d), the firm's supply function (y^s), and the profits π^*.

b) Find the demands for goods x and y for each consumer.

c) Find the Walrasian equilibrium, this is, the quantities consumed by each agent for each good, the input quantity used (x), and the firm's production (y).

Solution:
3) The firm's demand function for the input will be denoted as x^d, and the firm's supply is y^s. The optimization problem is

$$\max_{(x,y)} \Pi = p_x x + p_y y, \quad \text{s.t. } y \leq \sqrt{-x},\ x \leq 0.$$

Thus, the firm's problem reduces to (the solution is certainly on the boundary)

$$\max_{(x,y)} \Pi = p_x x + p_y \sqrt{-x}, \text{ s.t. } x \leq 0.$$

The Lagrangian is given by

$$\mathcal{L} = p_x x + p_y \sqrt{-x} - \lambda x.$$

The FOCs provide

$$\frac{\partial \mathcal{L}}{\partial x} = 0 \implies p_x - \frac{p_y (-x)^{-1/2}}{2} - \lambda = 0$$

and

$$\lambda x = 0.$$

If $\lambda = 0$, then $x < 0$. If $\lambda > 0, x = 0$. We are only interested in the first case.

$$\begin{aligned} x^d &= -\left(\frac{p_y}{2p_x}\right)^2 \\ y^s &= -\sqrt{x^d} = \frac{p_y}{2p_x} \\ \Pi^* &= \frac{p_y^2}{4p_x}. \end{aligned}$$

Regarding the consumers, we must solve

$$\begin{aligned} &\max_{x_i, y_i} u(x_i, y_i) \\ &\text{s.t. } p_x x_i + p_y y_i = p_x \omega_x + \theta_i \Pi^*. \end{aligned}$$

Given that the utility functions are Cobb-Douglas type

$$x_1^d = \frac{1}{2}\left(\frac{3p_x + \Pi^*/2}{p_x}\right)$$
$$y_1^d = \frac{1}{2}\left(\frac{3p_x + \Pi^*/2}{p_y}\right)$$
$$x_2^d = \frac{2}{3}\left(\frac{2p_x + \Pi^*/2}{p_x}\right)$$
$$y_2^d = \frac{1}{3}\left(\frac{2p_x + \Pi^*/2}{p_y}\right).$$

Normalizing $p_x = 1$ and replacing the expression of Π,

$$x_1^d = \frac{1}{2}\left(3 + p_y^2/8\right)$$
$$y_1^d = \frac{1}{2}\left(3/p_y + p_y/8\right)$$
$$x_2^d = \frac{2}{3}\left(2 + p_y^2/8\right)$$
$$y_2^d = \frac{1}{3}\left(2/p_y + p_y/8\right).$$

Finally, applying Walras' law for this context:

$$y_1^d + y_2^d - y^s = 0$$

$$\frac{1}{2}\left(3/p_y + p_y/8\right) + \frac{1}{3}\left(2/p_y + p_y/8\right) - \frac{p_y}{2} = 0$$
$$\frac{24 + p_y^2}{16p_y} + \frac{16 + p_y^2}{24p_y} = \frac{p_y}{2}$$
$$\frac{104 + 5p_y^2}{48p_y} = \frac{p_y}{2}$$
$$104 + 5p_y^2 = 24p_y^2$$
$$104 = 19p_y^2$$

we obtain $p_y = 2.3$. Thus,

$$(x_1^*, y_1^*) = (1.8, 0.8),\ (x_2^*, y_2^*) = (1.8, 0.4),\ x^{*d} = 1.4, \text{ and } y^{*s} = 1.2.$$

Exercise 4 (4 points). In the Economics Department at PUCP, the only seller of algorithms (x) , Manuel, faces a demand curve given by $x = a - bp$, where $a, b > 0$ and p is the price per algorithm sold. We assume that an algorithm is a perfectly divisible good, so $x \in \mathbb{R}_+$. Manuel has a quadratic cost function $C(x) = 2x^2 + 10x + \bar{c}$ in the number of algorithms sold ($\bar{c} > 0$ is a parameter).

1. Find the quantity of algorithms that Manuel sells (x^m) and the price at which he sells them (p^m). Remember that Manuel, given the context, operates as a monopolist. Your answer will depend on a and b. For your answer to make sense ($x^m \geq 0$), which is the relation that a and b must satisfy?

2. What happens with x^m and p^m if b increases?

3. If the fixed cost changes to $2\bar{c}$, do any of the previous answers changes? Why?

Solution:
4.1) Solving $\max_{x\geq 0}\left\{\left(\dfrac{a-x}{b}\right)x - C(x)\right\}$, we obtain $x^m = \dfrac{a-10b}{2+4b} > 0$ and $p^m = \dfrac{a}{b} - \dfrac{x^m}{b} = \dfrac{4ab+a+10b}{2b(2b+1)} > 0$, provided that $a > 10b$.

4.2) We directly compute

$$\frac{dx^m}{db} = -\frac{4a+20}{(2+4b)^2} < 0.$$

Finally,

$$\frac{dp^m}{db} = -\frac{8ab^2+20b^2+4ab+a}{2b^2(2b+1)^2} < 0.$$

4.3) Nothing happens, fixed costs won't change x^m or p^m, the only change is Π^m, but we never asked for Π^m in the previous items.

4.3) Answers don't change, fixed costs won't change the monopolist optimal level of production (so the optimal price neither).

Exercise 5 (2 points). Give an example of a **weak** Pareto optimal allocation which is not a Pareto optimum. Consider only continuous and monotone preferences.

Solution:
5) Consider $u_1(x_1, y_1) = f(x_1 + y_1)$, with $f' > 0$ and $u_2(x_2, y_2) = \min\{ax_2, ay_2\}$, $a > 0$. Then, $\{(0,0), (2.5, 3)\}$ is a weak Pareto optimum but not a Pareto optimum. Other admissible solutions are, for instance, $u_1(x_1, y_1) = f(x_1 + y_1)$ and $u_2(x_2, y_2) = c\max\{x_2, y_2\}$, with $c > 0$ or both $\max\{\cdot, \cdot\}$.

Viernes económicos (2 points)

a) El Perú es el **segundo** país líder en el mundo en la emisión de créditos de carbono forestal.

b) COFIDE, al ser el Banco de Desarrollo del Perú, busca financiar proyectos que no tengan solo un impacto **económico**, sino que también tengan un impacto **sostenible**.