FACULTY OF SOCIAL SCIENCES
ECONOMICS MAJOR

Solutions to First Qualified Exercises Session

Microeconomics 2
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Exercise 1. [5 points].

1. Provide the following definitions for a 2×2 economy. [1 point each one].
   a) Pareto optimal allocation.
   b) Walrasian equilibrium.
2. Analyze the truth or falsity of the following statements. Justify your answers. [1.5 points each one].
   a) In a pure exchange economy, if preferences are monotone, then the allocation of the Walrasian equilibrium is a Pareto Optimal allocation.
   b) When preferences are continuous, monotonic and concave, the Second Welfare Theorem holds.

1.a) An allocation $\mathbf{x} = (\mathbf{x}_1, \cdots, \mathbf{x}_I) \in \mathbb{R}_+^{IL}$ is Pareto Optimal if

$$\nexists\, \mathbf{x}' \in \mathbb{R}_+^{IL} : \sum_{i=1}^{I} \mathbf{x}'_i \leq \overline{\boldsymbol{\omega}},\ \forall\, i\ \mathbf{x}'_i \succeq_i \mathbf{x}_i \quad \wedge \quad \mathbf{x}'_{i_0} \succ_{i_0} \mathbf{x}_{i_0},\ i_0 \in \{1, ..., I\}.$$

1.b) A Walrasian Equilibrium is an allocation $\mathbf{x}^*$ and a price vector $\mathbf{p}^* \in \mathbb{R}_+^L$ such that:

1. $\mathbf{x}_i^* \in B(\mathbf{p}^*, \mathbf{p}^* \cdot \boldsymbol{\omega}_i)$, and $\mathbf{x}_i^* \succeq_i \mathbf{x}_i$, $\forall\, \mathbf{x}_i \in B(\mathbf{p}^*, \mathbf{p}^* \cdot \boldsymbol{\omega}_i)$.
2. $\sum_{i=1}^{I} \mathbf{x}_i^*(\mathbf{p}^*) = \sum_{i=1}^{I} \boldsymbol{\omega}_i$.

2.a) Truce since $\succeq_i$ monotone implies $\succeq_i$ locally non satiated. Indeed, recall that, $\succeq$ is locally non satiated over $X = \mathbb{R}^L$ if for every $x \in X$ and $\epsilon > 0$, there exists $y \in \mathcal{B}(x, \epsilon) = \{z \in \mathbb{R}^L : ||x - z|| = \sqrt{\sum_{i=1}^{L}(x_i - z_i)^2} < \epsilon\}$ such that $y \succ x$. Thus, consider $\mathbf{z} = \mathbf{x} + \sqrt{\frac{\epsilon}{2L}}\mathbf{1}_L$. Then, $\mathbf{z} \succ \mathbf{x}$ and $||\mathbf{z} - \mathbf{x}||_2 < \epsilon$.

The conclusion follows from First Welfare Theorem. The proof that locally non satiated is implied by monotonicity is not required.

2.b) False, preferences must be convex. Consider for instance

$$
\begin{aligned}
u_1(x, y) &= \max\{\min\{x, y/2\}, \min\{x/2, y\}\} \\
u_2(x, y) &= \min\{x, y\}.
\end{aligned}
$$

There is no equilibrium with these preferences. **Why?** (Homework).

Exercise 2. [4 points]. Consider two individuals in a pure exchange economy whose indirect utilities are

$$
\begin{aligned}
v_1(p_1, p_2, w) &= \ln w - a \ln p_1 - (1-a) \ln p_2 \\
v_2(p_1, p_2, w) &= \ln w - b \ln p_1 - (1-b) \ln p_2, \;\; a, b \in (0,1)
\end{aligned}
$$

Endowments are $\omega_1 = (1,1)$ and $\omega_2 = (1,1)$. Obtain the prices that clear the market. *Hint*: apply Roy's identity.

Solution: Roy's identity gives

$$
x_i^*(p) = -\frac{\dfrac{\partial v}{\partial p_i}}{\dfrac{\partial v}{\partial w}}.
$$

Thus,

$$
\begin{aligned}
x_{11}(p_1, p_2, w) &= \frac{aw}{p_1} \\
x_{21}(p_1, p_2, w) &= \frac{(1-a)w}{p_2} \\
x_{12}(p_1, p_2, w) &= \frac{bw}{p_1} \\
x_{22}(p_1, p_2, w) &= \frac{(1-b)w}{p_2}.
\end{aligned}
$$

Given that the income of individual 1 is

$$
p \cdot \omega_1 = p_1 + p_2,
$$

and the one of individual 2 is

$$
p \cdot \omega_2 = p_1 + p_2.
$$

it follows that

$$
\begin{aligned}
x_{11}(p_1, p_2) &= \frac{a(p_1 + p_2)}{p_1} \\
x_{21}(p_1, p_2, w) &= \frac{(1-a)(p_1 + p_2)}{p_2} \\
x_{12}(p_1, p_2, w) &= \frac{b(p_1 + p_2)}{p_1} \\
x_{22}(p_1, p_2, w) &= \frac{(1-b)(p_1 + p_2)}{p_2}.
\end{aligned}
$$

Applying Walras' Law to solve for the price ratio,

$$\frac{a(p_1+p_2)}{p_1}+\frac{b(p_1+p_2)}{p_1}=2.$$

Thus,

$$\frac{p_2^*}{p_1^*}=\frac{2-a-b}{a+b}.$$

Finally,

$$\frac{d}{da}(p_2^*/p_1^*)=-\frac{2}{(a+b)^2}<0.$$

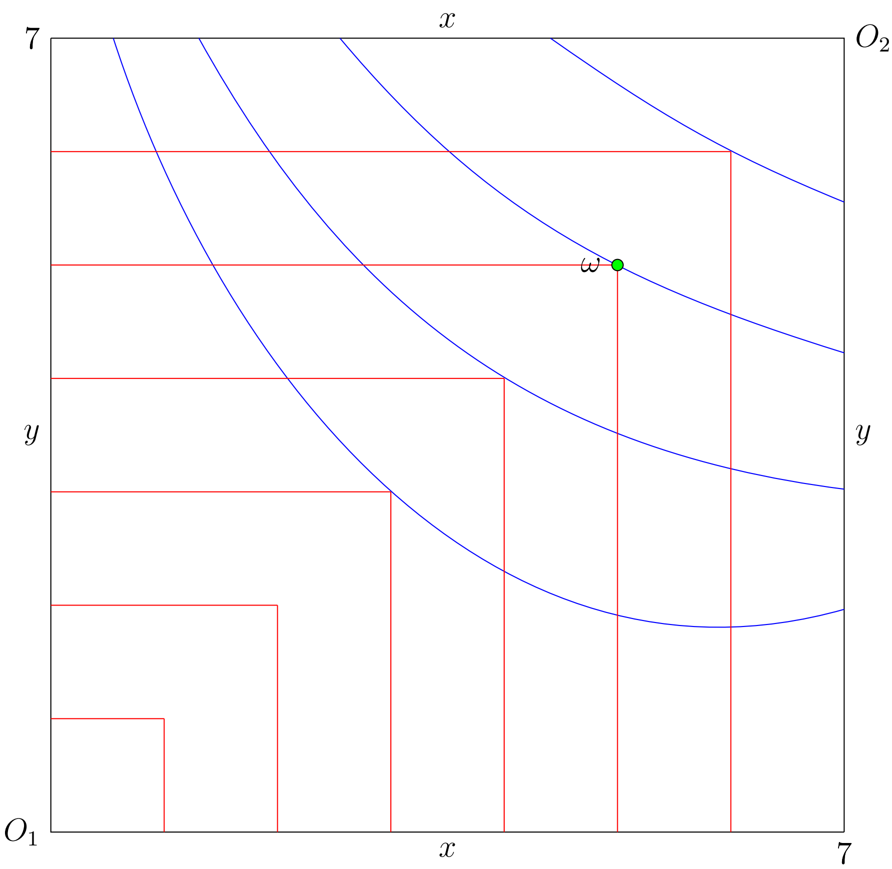
This means that if a increases, x_2 becomes relatively cheaper. This is beacuse consumer 1 values more good 1 than 2: he is ready to pay more for good 1.

Exercise 3. [5 points]. Consider a 2×2 economy where preferences are represented by the following utility functions: $u_1(x_{11},x_{21})=x_{11}^{\theta}x_{21}^{1-\theta}$, $\theta\in(0,1)$ and $u_2(x_{12},x_{22})=\min\{x_{12},x_{22}\}$, with endowments $\boldsymbol{\omega}_1=(5,5)$ and $\boldsymbol{\omega}_2=(2,2)$.

a) In the framework of the Edgeworth box, represent the endowments and some indifference curves (at least one for each consumer). Assume $\theta=1/3$ for this item. [2.5 points].

b) Obtain the Walrasian equilibrium in terms of θ. and analyze how the ratio of prices changes with respect to θ and interpret. [2.5 points].

a) See the following Figure.



b) Optimal demands are given by

$$
\begin{aligned}
x_{11}^* &= \frac{\theta(5p_1+5p_2)}{p_1}\\
x_{21}^* &= \frac{(1-\theta)(5p_1+5p_2)}{p_2}\\
x_{12}^* &= \frac{2p_1+2p_2}{p_1+p_2} = 5\\
x_{22}^* &= \frac{2p_1+2p_2}{p_1+p_2} = 5.
\end{aligned}
$$

Clearing the first market, we obtain

$$\frac{\theta(5p_1+5p_2)}{p_1} + 2 - 7 = 0.$$

Hence,

$$5\theta + 5\theta\left(\frac{p_2}{p_1}\right) = 5.$$

Finally,

$$\frac{p_2}{p_1} = (1-\theta)/\theta.$$

If θ increases, p_2/p_1 decreases. This is because consumer 1 values more godd 1 and is ready to pay more for it. Finally, it is easy to check that the final consumption will be the initial endowments. This is because $\boldsymbol{\omega}$ was already a Pareto Optimal allocation supported by the price vector already mentioned.

Exercise 4. [5 points]. Consider an economy with two consumers whose preferences and endowments are given by

$$
\begin{aligned}
u_1(x_{11}, x_{21}) &= x_{11}^{1/2}x_{21}^{1/2}, \quad \boldsymbol{\omega}_1 = (a, 1)\\
u_1(x_{12}, x_{22}) &= x_{12}^{4/5}x_{22}^{1/5}, \quad \boldsymbol{\omega}_2 = (1, a).
\end{aligned}
$$

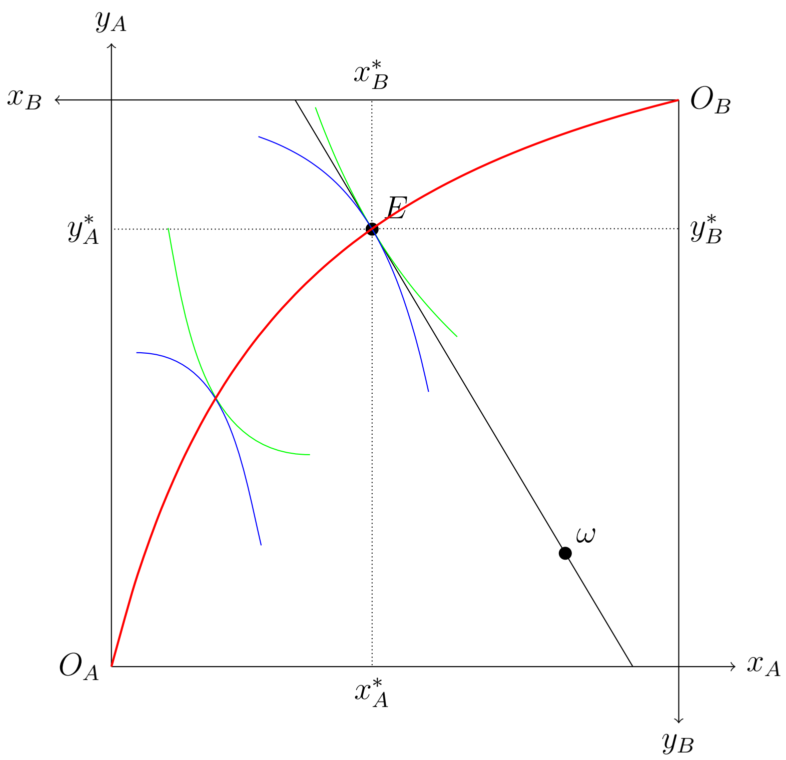
Here $a > 1 > 0$.

1. Set the optimization problem to find Pareto Optimal allocations. [1 point].
2. Plot the Pareto set in the Edgeworth box. [2 points].
3. Find all Pareto Optimal allocations. Your answer must be a subset of the Edgeworth box. This is: $(x_{11}, x_{21}) \in \mathcal{P} \subset [0, \omega_1] \times [0, \omega_2]$. [2 points].

1) The optimization problem is

$$
\begin{aligned}
\max_{x_{11},x_{21},x_{12},x_{22}} \quad & u_1(x_{11}, x_{21}) = x_{11}^{1/2}x_{21}^{1/2}\\
\text{s. t. } & u_2(x_{12}, x_{22}) = x_{12}^{4/5}x_{22}^{1/5} \geq \overline{u}\\
& x_{11} + x_{12} \leq a + 1\\
& x_{21} + x_{22} \leq 1 + a\\
& x_{\ell i} \geq 0.
\end{aligned}
$$

2) See the following figure:



3) The Pareto set is

$$x_{21} = \frac{4(1+a)x_{11}}{1+a+3x_{11}}, \; x_{11} \in [0, 1+a].$$

It if very important to note the concavity of $f(x_{11}) = \frac{4(1+a)x_{11}}{1+a+3x_{11}}, \; x_{11}$. This follows directly from the fact that $f''(x_{11}) < 0$. Moreover, $f' > 0$. These facts are key to draw correctly the Pareto set. Otherwise, you can use (several) indifference curves.

Lima, September 9, 2024.