

The Market and the Jungle*

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Abstract We study how different institutions governing property rights and economic transactions shape occupational choices and welfare. Individuals in our society choose occupations that determine their production and consumption opportunities, anticipating either anonymous, voluntary market exchange supported by public enforcement (*the market*), or allocations through a random coercive hierarchy (*the jungle*) without monetary compensation. Holding the population and occupational opportunities fixed, we establish conditions for occupational equilibria to exist in the market and in the jungle, and compare their performance. At a common occupational profile, markets maximize aggregate surplus, whereas selected hierarchical allocations are Pareto efficient with monetary holdings fixed but need not maximize surplus. Endogenous occupational choices introduce an additional source of institutional differences. Our main result shows that a market occupational equilibrium maximizes aggregate surplus over occupations and trades when it is *perfectly competitive*, i.e., when unilateral occupational deviations leave equilibrium prices unchanged. Such an equilibrium delivers aggregate payoffs at least as high as those of every jungle occupational equilibrium. We also establish an individual welfare comparison that does not require this price-invariance condition: among identical individuals at a symmetric jungle occupational equilibrium, everyone weakly prefers any market occupational equilibrium. Our analysis places the institutional foundations of exchange, including property rights, at the center of the welfare comparison. It also distinguishes collective performance from individual incentives, providing a framework for studying the appeal of coercive institutions that survive alongside markets.

Keywords: jungle, market, serial dictatorship, perfect competition, occupational equilibrium, welfare.

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1. Introduction

In this paper, we compare the allocation that obtains from a market economy—for short, *the market*—with the allocation that obtains from an alternative mechanism based on the use of coercion—for short, *the jungle*. In the market, an allocation results from voluntary transactions among individuals based on prices. In the jungle, an allocation results from the exercise of coercion in a hierarchy of power similar to that of a serial dictatorship. Individuals choose occupations in anticipation of the allocation rules of each economy. Our concern is how these rules shape both what individuals choose to do and the welfare they attain.

A key feature of our model is that each individual is characterized by the set of occupations she has access to in each economy. These occupational opportunities, moreover, may or may not be the same. Occupations determine what individuals in this society can do as consumers and as producers, as in [Makowski and Ostroy \(1995\)](#): an occupation specifies a valuation function and a set of feasible trades. Preferences are quasilinear in a numeraire (money). In the market, an allocation is reached concurrently with commodity prices via anonymous, decentralized trading and enforceable monetary transfers. In the jungle, more powerful individuals can decide the trading behavior of less powerful ones, as in [Piccione and Rubinstein \(2007\)](#). Thus, these institutions differ in both the way transactions are enforced and the scope for compensating individuals for their exchange activities.

Individuals choose their occupations before trading takes place in the market and before the power hierarchy is realized in the jungle. The hierarchy is drawn uniformly from all possible power orderings, independently of occupational choices. This captures uncertainty about the exercise of coercive power; individuals do not invest in their positions in the hierarchy, and their trading opportunities are not related to their coercive capacities. An occupational equilibrium is a profile at which no individual gains by changing her occupation, anticipating the resulting market payoff or expected jungle payoff.

We derive two main results.

First, we consider a scenario in which the profile of occupations is the same in the market and in the jungle. We show that under those circumstances, individuals are collectively weakly made better off in the market than in the jungle, and under weak assumptions, strictly better off. This result rests upon well-known intuitions behind the properties of competitive equilibrium. At the selected occupational profile, both market equilibrium and jungle equilibrium are locally Pareto efficient, but with quasilinear preferences and monetary transfers, only the market equilibrium maximizes aggregate payoffs, understood as the sum of valuations. Because the jungle lacks a public institution governing property rights and enforcing contracts, money supply is not available to compensate individuals and induce a surplus-enhancing allocation.

Second, allowing individuals to freely choose their occupations before transactions in the market and before power uncertainty resolves in the jungle, so that the equilibrium profile of occupations in both economies may differ, we show that individuals are collectively made better off in the market than in the jungle under two conditions: (i) every potential occupation in the jungle is available in the market, and (ii) the market is perfectly competitive, in the sense that prices cannot be manipulated by any single individual, neither via trading decisions nor via occupational decisions. Thus, when these two conditions are present, any occupational

equilibrium in the market collectively outperforms any occupational equilibrium in the jungle.¹

Our first result parallels traditional welfare theorems in an economy with transferable utility. Our second result goes beyond, and provides a template to understand situations in which groups of individuals may opt out of the market. In particular, we interpret failures of condition (i) as circumstances in which due to social exclusion, stigma, or force, some occupations are only available in the jungle. We interpret failures of condition (ii) as corresponding to oligopolistic or monopolistic market structures in the market economy, in a broader sense than the standard interpretation in industrial organization models. In particular, the jungle economy may be preferable to the market economy for a group of individuals if certain occupations are available for them in the jungle but not in the market, or if prices can be rigged by consumption and/or production decisions to not fully reflect competitive forces, or if some individuals have occupations available to them that give them means to influence market prices. Thus, our model provides a flexible framework to explain, in terms of general equilibrium, the persistence of non-anonymous trading under duress in spite of the superiority of the market mechanism.

Competitive markets in economic models are both an idealized representation of the complex process of allocating resources in a market economy with well-defined property rights and, via the welfare theorems, a representation of the ideal solution of the resource allocation problem. The state is often counterposed to the market as either, positively, a source of security in property rights, impartial enforcement of contracts, and a solution to potential market failures, or, negatively, as a source of deviations from the ideal market allocation due to unjustified government intervention. This to some extent betrays the origins of the discipline in the preoccupation with fiscal affairs and public policy of the classical political economists. In line with [Basu \(2003\)](#), we think instead of the economy as deeply embedded in social institutions that constrain the full gamut of individual behaviors—including occupational and trading decisions—and the working of prices. These social institutions operate via formal state power or via social norms, traditions and the use of force. Indeed, examples of social exclusion due to ethnic, religious, or language differences are ubiquitous, though their importance in different economies can vary hugely. Exclusion, seizure of output and compelled labor whether or not mediated by the state are examples of what we interpret as primary conditions for the emergence of an alternative economic system opposing the market, which we call the jungle.

We eschew the traditional discussion of market shortcomings as “market failures.” In particular, we do not counterpose the market to the idealized intervention of a benevolent state that corrects market shortcomings. Rather, we are interested in comparing allocations predicted by the market mechanism with those dictated by the alternative allocation by coercive force, and ask under what circumstances the second may afford some individuals a better prospect *ex ante*. This allows us to pin down, in the language of general equilibrium expanded to include occupational choices, precise conditions for the coexistence of the market and the jungle. Thus, our paper highlights not just the importance of price-taking behavior, including in occupational choices, but also the role of market exclusion via social norms, traditions, and state regulations.

¹This aggregate comparison leaves open whether particular individuals prefer the jungle. We obtain an individual comparison under a different condition, without requiring perfect competition. This should be of independent interest, which we discuss at the end of [Section 5.2](#).

The remainder of the paper is organized as follows. [Section 2](#) describes the model of our society, one in which individuals can choose whether to join the market or the jungle and what occupation to select. [Section 3](#) describes in detail the workings of the jungle economy, and provides conditions for the existence of an occupational equilibrium in the jungle. Turning to the market economy, [Section 4](#) provides conditions for the existence of an occupational equilibrium in the market. At any occupational profile, a local market equilibrium exists and is Pareto efficient. Likewise, at any occupational profile, a serial dictatorship procedure selects local jungle equilibria that are Pareto efficient when monetary holdings are fixed. Passing from these allocations to occupational equilibrium requires additional care. We illustrate some of these difficulties with a few examples. [Section 5](#) compares the outcomes of both economies and provides our two main results described above.

2. Setting

We start with a general description of a society as a finite group of individuals who make occupational choices and engage in production and consumption activities. Some of these individuals interact in the *market*, some of them interact in the *jungle*. Individuals have a choice as to which economy they belong to. In this paper, to simplify the analysis we take the initial participation choices to be irrevocable.² These economies operate under different rules, which we explain below. The crucial distinction between them lies in their enforcement of economic transactions: anonymous exchange in the market relies on public enforcement of property rights and contracts; the coordination of economic activity in the jungle relies instead on private enforcement by a hierarchy.

The set of individuals who belong to the market is $\mathcal{M}$, the set of individuals who belong to the jungle is $\mathcal{J}$. We assume $\mathcal{M}$ and $\mathcal{J}$ to be disjoint and let $\mathcal{S} = \mathcal{M} \cup \mathcal{J}$ denote the set of all individuals in our society. In explaining our framework, we treat market and jungle as nonempty. With endogenous participation, it is possible that $\mathcal{M} = \emptyset$ or that $\mathcal{J} = \emptyset$ (but both cannot be empty). We are particularly interested in the case where these two economies coexist, i.e., $\mathcal{M} \neq \emptyset$ and $\mathcal{J} \neq \emptyset$ simultaneously.

We assume that individuals have convex, quasilinear preferences over L non-monetary commodities and money. Think of the L commodities as indexing all the goods and services that could be offered in this society. Some goods may be actively traded mostly in the market, others mostly in the jungle, and some in both economies. All individuals in this society act both as consumers and entrepreneurs. To capture this in a flexible way, we take a cue from [Makowski and Ostroy \(1995\)](#) and let $z_i \in \mathbb{R}^L$ be a *trade* for each $i \in \mathcal{S}$, with the sign convention that treats $z_{i\ell} > 0$ as the amount of good ℓ being purchased and $z_{i\ell} < 0$ as the amount being sold by individual i . Inequalities between vectors are component-wise throughout.

Each individual $i \in \mathcal{S}$ is endowed with a nonempty set of *occupations* Θ_i .³ Occupations together with trades determine individual payoffs. A trade z_i and a money holding $m_i \in \mathbb{R}$

²Our focus on entry decisions highlights individual agency, but entry and exit also face frictions. [Melnikov et al. \(2025\)](#) show that criminal organizations in El Salvador restrict individuals' mobility and labor-market options; [Sviatschi \(2022\)](#) documents the persistence of criminal career paths.

³One interpretation: there are K productive activities available to i in this society, and θ_{ik} is the share of time that i devotes to activity k . So Θ_i is the simplex $\{\theta_i \in \mathbb{R}_+^K : \sum_{k=1}^K \theta_{ik} \leq 1\}$. Nothing below depends on this reading.

give individual i a utility of $v_i(z_i, \theta_i) + m_i$, where

$$v_i: \mathbb{R}^L \times \Theta_i \rightarrow \mathbb{R} \cup \{-\infty\}.$$

The cardinal normalization is part of the primitives and is fixed across occupations, so payoffs are comparable across the occupational choices available to an individual. Money is the unit of account in both economies, although, as we shall see later, only the market trades the numeraire.

An occupation $\theta_i \in \Theta_i$ also determines the set of individually feasible trades

$$Z_i(\theta_i) \equiv \{z_i \in \mathbb{R}^L : v_i(z_i, \theta_i) > -\infty\}.$$

$Z_i(\theta_i)$ records what individual i can consume and produce at her given occupational choice. The value $-\infty$ outside $Z_i(\theta_i)$ encodes infeasibility. The family of induced valuation functions for the non-monetary commodities is $\{v_i(\cdot, \theta_i) : \theta_i \in \Theta_i\}$. We consider the mapping $\theta_i \mapsto v_i(\cdot, \theta_i)$ to be injective, so the parameter θ_i and the induced valuation function identify the same occupation.

For the remainder of the paper, the following is maintained.

Assumption 1. For each $i \in \mathcal{S}$, assume that

- (a) the set of occupations Θ_i is a nonempty, compact and convex subset of a finite-dimensional normed vector space;
- (b) the correspondence $Z_i: \Theta_i \rightrightarrows \mathbb{R}^L$ is continuous with nonempty, compact, and convex values, and moreover $\mathbf{0} \in Z_i(\theta_i)$, for all $\theta_i \in \Theta_i$;
- (c) the function $v_i: \mathbb{R}^L \times \Theta_i \rightarrow \mathbb{R} \cup \{-\infty\}$ is continuous on $\{(z, \theta_i) : z \in Z_i(\theta_i), \theta_i \in \Theta_i\}$.
- (d) Finally, $\mathbf{0} \in \text{int} \left\{ \sum_{i \in \mathcal{S}} Z_i(\theta_i) \right\}$, for all $\boldsymbol{\theta} = (\theta_i) \in \boldsymbol{\Theta} = \times_{i \in \mathcal{S}} \Theta_i$.

Most of the conditions stated in [Assumption 1](#) are standard in parametric optimization problems, and we will employ some of these tools in our analysis. We point out that, since individuals in our society have convex, quasilinear preferences over commodities and the numeraire, $v_i(\cdot, \theta_i)$ is concave on $Z_i(\theta_i)$, for all $i \in \mathcal{S}$. We explain the role our conditions play in due course.

In our model, individuals choose occupations in the market and in the jungle. The initial participation decision, whether to belong to the jungle or to the market, is influenced by the occupations an individual has available in each of these economies, which may differ due to formal regulations in the market, and customs or social norms in the market and in the jungle. To capture this, we let $\Theta_i^{\mathcal{M}} \subseteq \Theta_i$ and $\Theta_i^{\mathcal{J}} \subseteq \Theta_i$ denote the (nonempty) sets of occupations available for $i \in \mathcal{S}$ if she chooses to join the market or the jungle, respectively. In some contexts, it is possible that $\Theta_i^{\mathcal{M}} = \Theta_i^{\mathcal{J}} = \Theta_i$. Other contexts may allow individual i to attain some occupations in the jungle that are not available for her in the market, and vice versa. Our framework is sufficiently flexible to capture these possibilities. Let $\boldsymbol{\Theta}^{\mathcal{M}} = \times_{i \in \mathcal{M}} \Theta_i^{\mathcal{M}}$ and $\boldsymbol{\Theta}^{\mathcal{J}} = \times_{i \in \mathcal{J}} \Theta_i^{\mathcal{J}}$. An *occupational profile* for the economy $\mathcal{E} \in \{\mathcal{M}, \mathcal{J}\}$, denoted $\boldsymbol{\theta}^{\mathcal{E}} = (\theta_i^{\mathcal{E}}) \in \boldsymbol{\Theta}^{\mathcal{E}}$, specifies an occupational choice for each individual in the given economy. We

impose [Assumption 1](#) on each nonempty economy as well, replacing $\mathcal{S}$ and Θ_i by $\mathcal{E}$ and $\Theta_i^\mathcal{E}$, respectively.

A trade profile $\mathbf{z} = (z_i)_{i \in \mathcal{E}}$ for the economy $\mathcal{E} \in \{\mathcal{M}, \mathcal{J}\}$ is a collection of trades, one for each member of the economy. A first requirement for a trade profile to be feasible in $\mathcal{E}$ is that each trade is individually feasible given the occupational choices selected by each member of that economy. A second requirement is that trades among all individuals balance out. We impose these two conditions throughout.

Definition 1. Given an occupational profile $\theta^\mathcal{E} \in \Theta^\mathcal{E}$ for the economy $\mathcal{E} \in \{\mathcal{M}, \mathcal{J}\}$, a trade profile $\mathbf{z} = (z_i)_{i \in \mathcal{E}}$ is feasible at $\theta^\mathcal{E}$ if the following two conditions are satisfied:

- (a) for all $i \in \mathcal{E}$, $z_i \in Z_i(\theta_i^\mathcal{E})$;
- (b) $\sum_{i \in \mathcal{E}} z_i = \mathbf{0}$.

The clearing condition in [Definition 1\(b\)](#) is imposed within each economy, so the market and the jungle clear separately, with no trade between them. Let $\mathbf{Z}(\theta^\mathcal{E})$ denote the set of all trade profiles for $\mathcal{E}$ that are feasible at $\theta^\mathcal{E} \in \Theta^\mathcal{E}$. We state the next simple observation for completeness.

Lemma 1. Let $\mathcal{E} \in \{\mathcal{M}, \mathcal{J}\}$ be a nonempty economy. For any occupational profile $\theta^\mathcal{E} \in \Theta^\mathcal{E}$, the set of feasible trade profiles $\mathbf{Z}(\theta^\mathcal{E}) \subseteq \mathbb{R}^{|\mathcal{E}| \times L}$ is nonempty, compact and convex.

Proof. Let $F \equiv \{\mathbf{z} = (z_i) \in \mathbb{R}^{|\mathcal{E}| \times L} : \sum_i z_i = \mathbf{0}\}$ represent the trading feasibility constraint in $\mathcal{E}$. For any occupational profile $\theta^\mathcal{E} = (\theta_i^\mathcal{E}) \in \Theta^\mathcal{E}$, we can express the set of feasible trades as

$$\mathbf{Z}(\theta^\mathcal{E}) = \left[\times_{i \in \mathcal{E}} Z_i(\theta_i^\mathcal{E}) \right] \cap F.$$

Clearly, $\mathbf{Z}(\theta^\mathcal{E})$ is nonempty, either by [Assumption 1\(b\)](#) or [Assumption 1\(d\)](#). Further, by [Assumption 1\(b\)](#), $Z_i(\theta_i^\mathcal{E})$ is compact and convex for all $i \in \mathcal{E}$, and thus so is the product set $\times_{i \in \mathcal{E}} Z_i(\theta_i^\mathcal{E})$. Since the feasibility constraint F is closed and convex, it follows that $\mathbf{Z}(\theta^\mathcal{E})$ is compact and convex. $\square$

To evaluate the performance of these economies, once occupational choices have been made, we employ a Pareto efficiency criterion with respect to the non-monetary commodities.

Definition 2. Given an occupational profile $\theta^\mathcal{E} \in \Theta^\mathcal{E}$ for the economy $\mathcal{E} \in \{\mathcal{M}, \mathcal{J}\}$, a trade profile $\mathbf{z} = (z_i)_{i \in \mathcal{E}}$ is Pareto efficient at $\theta^\mathcal{E}$ if it is feasible at $\theta^\mathcal{E}$, i.e., $\mathbf{z} \in \mathbf{Z}(\theta^\mathcal{E})$, and if there is no other trade profile $\hat{\mathbf{z}} = (\hat{z}_i)_{i \in \mathcal{E}}$, feasible at $\theta^\mathcal{E}$, such that $v_i(\hat{z}_i, \theta_i^\mathcal{E}) \geq v_i(z_i, \theta_i^\mathcal{E})$ for all $i \in \mathcal{E}$ and $v_j(\hat{z}_j, \theta_j^\mathcal{E}) > v_j(z_j, \theta_j^\mathcal{E})$ for at least some $j \in \mathcal{E}$.

Our notion of efficiency evaluates trade profiles keeping monetary holdings fixed. In the jungle, this is the relevant criterion, since monetary transfers are excluded: there is no anonymous institution enforcing contracts and organizing monetary supply. In the market, in contrast, allowing zero-sum monetary transfers gives a stronger notion of efficiency: under quasilinear preferences, a feasible allocation of commodities and money is Pareto efficient if and only if its trade profile maximizes the sum of individual valuations. Such a trade profile is also Pareto efficient in the sense of [Definition 2](#), but the converse need not hold. We distinguish between these two notions in our analysis.

3. The Jungle

A main distinction between the jungle and the market is the way each economy enforces contracts. The market supports anonymous exchange and enforceable monetary transfers. In the jungle economy, there exists no third-party or public institution governing property rights. Thus, the production and consumption of resources in the jungle are purely based on private enforcement without monetary compensations.

For this section, we take $\mathcal{J} \neq \emptyset$ to be given. The coordination of economic activity in the jungle is organized by a hierarchy (or strict power relation). We model a hierarchy as a bijection

$$h: \{1, 2, \dots, |\mathcal{J}|\} \rightarrow \mathcal{J}$$

that maps each position $n \in \{1, 2, \dots, |\mathcal{J}|\}$ in the hierarchy to the individual $h(n)$ who occupies it. Thus $h(1)$ is the most powerful individual, $h(2)$ the second most powerful, and so on. The interpretation is that, for any two positions $1 \leq n < m \leq |\mathcal{J}|$, individual $h(n)$ comes before $h(m)$ in the hierarchy h and therein controls, coercively if needed, any available resource that $h(m)$ may have. Let $\mathcal{H}(\mathcal{J})$ denote the set of all hierarchies on $\mathcal{J}$.

We assume that the exercise of coercive power is costless; for a discussion of this assumption, see [Piccione and Rubinstein \(2007\)](#). This is the counterpart of the frictionless exercise of property rights in a standard general equilibrium framework, which we will use to analyze the market economy. We record a power relation in the jungle as a bijection rather than as an abstract binary relation because it is the object over which we later randomize: a power relation is a permutation of $\mathcal{J}$, so a lottery over these hierarchies is a lottery over the finite set $\mathcal{H}(\mathcal{J})$.

3.1. Local Equilibrium in the Jungle

An equilibrium captures the idea of a stable social situation. In the jungle, consumption and production decisions are determined in a hierarchical way and enforced by private coercion. To capture these features of the workings of the jungle economy, we use a modified version of [Piccione and Rubinstein's \(2007\) jungle equilibrium](#).

Definition 3. Fix an occupational profile $\theta^\mathcal{J} \in \Theta^\mathcal{J}$ and a hierarchy $h \in \mathcal{H}(\mathcal{J})$. A trade profile $\mathbf{z}$ is a jungle equilibrium at $(\theta^\mathcal{J}, h)$ if it satisfies the following conditions:

- (a) $\mathbf{z}$ is feasible at $\theta^\mathcal{J}$, i.e., $\mathbf{z} \in \mathbf{Z}(\theta^\mathcal{J})$;
- (b) there do not exist two individuals $h(m), h(n) \in \mathcal{J}$, with $1 \leq m < n \leq |\mathcal{J}|$, and a feasible trade profile $\hat{\mathbf{z}} \in \mathbf{Z}(\theta^\mathcal{J})$, such that $v_i(\hat{z}_i, \theta_i^\mathcal{J}) = v_i(z_i, \theta_i^\mathcal{J})$ for all $i \in \mathcal{J}$, $i \neq h(m), h(n)$, and further

$$v_{h(m)}(\hat{z}_{h(m)}, \theta_{h(m)}^\mathcal{J}) > v_{h(m)}(z_{h(m)}, \theta_{h(m)}^\mathcal{J}).$$

We sometimes refer to a jungle equilibrium at given occupational profile and power relation as a *local jungle equilibrium*. Our definition of local jungle equilibria is stated in terms of utilities instead of trades. For an individual to “break an equilibrium,” she needs to find an alternative feasible trade profile that makes her strictly better off. She can change trades across multiple individuals in the jungle, but must do so in a way that leaves everyone else, except a

single weaker individual (in accordance with the hierarchy h), with the same utility level as in the original trade profile. Ours is stronger than the original one in [Piccione and Rubinstein \(2007\)](#), as their definition requires an individual $h(m)$ to change her trade and that of a single, weaker individual $h(n)$, leaving everyone else with the same trades. [Definition 3](#) fixes the other individuals only in *utility* terms: the trades of third parties, including those above $h(m)$ in the hierarchy, may be rearranged along their indifference sets, but no third party's welfare changes.

[Definition 3](#) can be interpreted as a *coercive blocking rule*, the power analogue of a blocking coalition: an individual $h(m)$ may costlessly propose any feasible reshuffle that strictly raises her own payoff and leaves every individual other than $h(m)$ and a single victim $h(n)$, ranked lower in the hierarchy (so $m < n$), at their original utility levels. This is a reduced form of costless coercion in a moneyless economy.

We now show that a local jungle equilibrium exists. Adapting the serial dictatorship procedure, we use the strict order given by the hierarchy h to define, recursively, a sequence of nested individual optimization problems. The critical property of these nested problems is that the solution set in one stage becomes the constraint set of the next stage. Since $|\mathcal{J}| < \infty$, the recursion is well defined and produces a nonempty set of feasible trade profiles, each of which constitutes a local jungle equilibrium.

Proposition 1. *For any occupational profile $\theta^{\mathcal{J}} \in \Theta^{\mathcal{J}}$ and any hierarchy $h \in \mathcal{H}(\mathcal{J})$, a local jungle equilibrium exists.*

Proof. Fix an occupational profile $\theta^{\mathcal{J}}$ and a hierarchy h , and consider the following recursion.

Initial Step. Define $\mathbf{Z}_{h(0)}(\theta^{\mathcal{J}}) \equiv \mathbf{Z}(\theta^{\mathcal{J}})$ as the constraint set of the most powerful individual. From [Lemma 1](#), $\mathbf{Z}_{h(0)}$ is nonempty and compact. Individual $h(1)$'s optimization problem is to choose a trade profile to maximize her utility among all feasible trade profiles, i.e.,

$$\max_z \left\{ v_{h(1)}(z_{h(1)}, \theta_{h(1)}^{\mathcal{J}}) : z \in \mathbf{Z}_{h(0)}(\theta^{\mathcal{J}}) \right\}.$$

Let $\pi_{h(1)}(\theta^{\mathcal{J}})$ denote the value of $h(1)$'s utility at any solution of the above problem. Define

$$\mathbf{Z}_{h(1)}(\theta^{\mathcal{J}}) \equiv \left\{ z \in \mathbf{Z}_{h(0)}(\theta^{\mathcal{J}}) : v_{h(1)}(z_{h(1)}, \theta_{h(1)}^{\mathcal{J}}) = \pi_{h(1)}(\theta^{\mathcal{J}}) \right\}$$

as the solution set of $h(1)$'s optimization problem. Clearly, from [Assumption 1\(c\)](#), $\mathbf{Z}_{h(1)}(\theta^{\mathcal{J}})$ is a nonempty, compact subset of $\mathbf{Z}_{h(0)}(\theta^{\mathcal{J}})$.

Induction Step. For $2 \leq n \leq |\mathcal{J}|$, consider the optimization problem of individual $h(n) \in \mathcal{J}$. She chooses a feasible trade profile to maximize her utility among all feasible trade profiles that leave every stronger individual $h(m)$, $m < n$, at a utility level equal to $\pi_{h(m)}(\theta^{\mathcal{J}})$. Thus, $h(n)$'s optimization problem is

$$\max_z \left\{ v_{h(n)}(z_{h(n)}, \theta_{h(n)}^{\mathcal{J}}) : z \in \mathbf{Z}_{h(n-1)}(\theta^{\mathcal{J}}) \right\}.$$

Let $\pi_{h(n)}(\theta^{\mathcal{J}})$ denote the value of $h(n)$'s utility at any solution of the above problem, and let

$$\mathbf{Z}_{h(n)}(\theta^{\mathcal{J}}) \equiv \left\{ z \in \mathbf{Z}_{h(n-1)}(\theta^{\mathcal{J}}) : v_{h(n)}(z_{h(n)}, \theta_{h(n)}^{\mathcal{J}}) = \pi_{h(n)}(\theta^{\mathcal{J}}) \right\}.$$

Clearly, the solution set $Z_{h(n)}(\theta^\mathcal{J})$ for individual $h(n)$'s optimization problem is a nonempty, compact subset of $Z_{h(n-1)}(\theta^\mathcal{J})$.

Terminal Step. Let $Z^*(\theta^\mathcal{J}, h) \equiv Z_{h(|\mathcal{J}|)}(\theta^\mathcal{J})$. This set is nonempty and compact: it is the last of the finitely many solution sets constructed above, each of which is the set of maximizers of a continuous function over a nonempty compact set.

We claim that any trade profile $z^* \in Z^*(\theta^\mathcal{J}, h)$ is a jungle equilibrium at $(\theta^\mathcal{J}, h)$. First, observe that the solution sets of the individual optimization problems in the above recursion are nested:

$$Z^*(\theta^\mathcal{J}, h) \equiv Z_{h(|\mathcal{J}|)}(\theta^\mathcal{J}) \subseteq \dots \subseteq Z_{h(1)}(\theta^\mathcal{J}) \subseteq Z_{h(0)}(\theta^\mathcal{J}) \equiv Z(\theta^\mathcal{J}).$$

Thus, $z^* \in Z^*(\theta^\mathcal{J}, h)$ is a feasible trade profile for $\theta^\mathcal{J}$. To obtain a contradiction, suppose there is another trade profile $\hat{z} \in Z(\theta^\mathcal{J})$ and two individuals $h(m), h(n)$, with $m < n$, such that $v_i(\hat{z}_i, \theta_i^\mathcal{J}) = v_i(z_i^*, \theta_i^\mathcal{J})$ for all $i \in \mathcal{J}$, $i \neq h(m), h(n)$, and moreover

$$v_{h(m)}(\hat{z}_{h(m)}, \theta_{h(m)}^\mathcal{J}) > v_{h(m)}(z_{h(m)}^*, \theta_{h(m)}^\mathcal{J}).$$

Since $h(m)$ is more powerful than $h(n)$, the trade profile $\hat{z}$ must leave every stronger individual $h(k)$, for $k < m < n$, at their original utility level. Hence the trade profile $\hat{z}$ satisfies the constraints defining each of the first $m - 1$ optimization problems in our recursion. It follows that $\hat{z} \in Z_{h(m-1)}(\theta^\mathcal{J})$. But this means $\hat{z}$ is feasible in $h(m)$'s optimization problem whose maximum value is $\pi_{h(m)}(\theta^\mathcal{J})$, so we must have

$$v_{h(m)}(\hat{z}_{h(m)}, \theta_{h(m)}^\mathcal{J}) \leq \pi_{h(m)}(\theta^\mathcal{J}) = v_{h(m)}(z_{h(m)}^*, \theta_{h(m)}^\mathcal{J}),$$

which is a contradiction. □

The construction in [Proposition 1](#) is an adaptation of the serial dictatorship procedure. In house allocation, agents pick in a fixed order among the objects that remain; an assignment is Pareto efficient if and only if it is the outcome of some serial dictatorship, and uniform randomization over priority orders coincides in distribution with the core from uniformly random endowments ([Abdulkadiroğlu and Sönmez, 1998](#)). Axiomatizations and extensions are given by [Pápai \(2000\)](#) and [Bade \(2020\)](#). The closest antecedent in a continuous environment is [Satterthwaite and Sonnenschein \(1981\)](#). Our serial dictators are constrained by their predecessors' *utility levels*, not their assigned positions; under the strict preferences of the housing model the distinction is moot. With endowments and consent the housing model with strict preferences yields top trading cycles ([Shapley and Scarf, 1974](#); [Roth and Postlewaite, 1977](#); [Roth, 1982](#)); removing both and replacing them with the power order gives the serial dictatorship above. [Balbuzanov and Kotowski \(2019\)](#) model rights to *exclude* rather than power to take; their exclusion core is a cooperative counterpart of the blocking rule behind the local jungle equilibrium of [Definition 3](#).

In proving [Proposition 1](#), we identify a family $Z^*(\theta^\mathcal{J}, h)$ of equilibrium trade profiles. Since individual preferences are convex, $Z^*(\theta^\mathcal{J}, h)$ is a convex set, within which the equilibria are utility-equivalent.

Corollary 1. Fix an occupational profile $\theta^\mathcal{J} \in \Theta^\mathcal{J}$ and a power relation $h \in \mathcal{H}(\mathcal{J})$.

- (a) $Z^*(\theta^\mathcal{J}, h)$ is convex.
- (b) If z^* and z^{**} are two local jungle equilibria in $Z^*(\theta^\mathcal{J}, h)$, then for every $i \in \mathcal{J}$, one has $v_i(z_i^*, \theta_i^\mathcal{J}) = v_i(z_i^{**}, \theta_i^\mathcal{J})$.
- (c) If $v_i(\cdot, \theta_i^\mathcal{J})$ is strictly concave for all $i \in \mathcal{J}$, then $Z^*(\theta^\mathcal{J}, h)$ is a singleton.

Proof. (a) We show by induction on the stages $1 \leq n \leq |\mathcal{J}|$ that each solution set $Z_{h(n)}(\theta^\mathcal{J})$ is convex, starting from $Z_{h(0)}(\theta^\mathcal{J}) = Z(\theta^\mathcal{J})$, which is convex by Lemma 1. Suppose now that $Z_{h(n-1)}(\theta^\mathcal{J})$ is convex. Let $z', z'' \in Z_{h(n)}(\theta^\mathcal{J})$, and let $z^\lambda = \lambda z' + (1 - \lambda)z''$, for any $\lambda \in [0, 1]$. Then $z^\lambda \in Z_{h(n-1)}(\theta^\mathcal{J})$, and using the concavity of $v_{h(n)}(\cdot, \theta_{h(n)}^\mathcal{J})$ we obtain

$$\begin{aligned} v_{h(n)}(z_{h(n)}^\lambda, \theta_{h(n)}^\mathcal{J}) &\geq \lambda v_{h(n)}(z'_{h(n)}, \theta_{h(n)}^\mathcal{J}) + (1 - \lambda) v_{h(n)}(z''_{h(n)}, \theta_{h(n)}^\mathcal{J}) \\ &= \pi_{h(n)}(\theta^\mathcal{J}). \end{aligned} \tag{1}$$

Since $\pi_{h(n)}(\theta^\mathcal{J})$ is the value of individual $h(n)$'s maximized utility over $Z_{h(n-1)}(\theta^\mathcal{J})$, equality holds in the above expression and thus $z^\lambda \in Z_{h(n)}(\theta^\mathcal{J})$.

(b) This is immediate from the recursion in the proof of Proposition 1.

(c) If any two trade profiles in $Z_{h(n)}(\theta^\mathcal{J})$ had different coordinates for individual $h(n)$, strict concavity of the valuation would make the inequality in Equation 1 strict for $0 < \lambda < 1$, which is impossible. Hence all trade profiles in $Z_{h(n)}(\theta^\mathcal{J})$ share the coordinate for $h(n)$. Since $Z_{h(n)}(\theta^\mathcal{J}) \subseteq Z_{h(n-1)}(\theta^\mathcal{J})$, it follows that the coordinates of $h(1), \dots, h(n-1)$ must also be the same, for any two trade profiles in $Z_{h(n)}(\theta^\mathcal{J})$. At the last stage of the recursion, every coordinate other than that of $h(|\mathcal{J}|)$ is fixed, and the valuation of the weakest individual is strictly concave. It follows that $Z^*(\theta^\mathcal{J}, h)$ is a singleton. $\square$

There may be additional local jungle equilibria outside $Z^*(\theta^\mathcal{J}, h)$, and these need not share the utility levels identified in Proposition 1. Therefore, the utility equivalence recorded in Corollary 1(b) is confined to the equilibrium class $Z^*(\theta^\mathcal{J}, h)$. We also point out that the uniqueness identified in Corollary 1(c) concerns the family $Z^*(\theta^\mathcal{J}, h)$. Strict concavity alone does not exclude local jungle equilibria outside that family. On the other hand, as we show in our next result, the local jungle equilibria in $Z^*(\theta^\mathcal{J}, h)$ are Pareto efficient at $\theta^\mathcal{J}$. This conclusion follows from the serial selection procedure; the local equilibrium definition alone does not guarantee Pareto efficiency. At each stage in the recursion used to obtain $Z^*(\theta^\mathcal{J}, h)$, an individual maximizes her utility among trade profiles preserving the utility levels obtained by individuals at the preceding stages.

Proposition 2. Given occupational profile $\theta^\mathcal{J} \in \Theta^\mathcal{J}$ and power relation $h \in \mathcal{H}(\mathcal{J})$, any local jungle equilibrium $z \in Z^*(\theta^\mathcal{J}, h)$ is Pareto efficient at $\theta^\mathcal{J}$.

Proof. Suppose $z \in Z^*(\theta^\mathcal{J}, h)$ is not Pareto efficient at $\theta^\mathcal{J}$. Since this trade profile is feasible for $\theta^\mathcal{J}$, there must be another trade profile $\hat{z} \in Z(\theta^\mathcal{J})$ such that $v_i(\hat{z}_i, \theta_i^\mathcal{J}) \geq v_i(z_i, \theta_i^\mathcal{J})$ for all $i \in \mathcal{J}$, with strict inequality for at least some individual in the jungle economy.

Let $h(n)$ be the most powerful individual in the hierarchy h with

$$v_{h(n)}(\hat{z}_{h(n)}, \theta_{h(n)}^\mathcal{J}) > v_{h(n)}(z_{h(n)}, \theta_{h(n)}^\mathcal{J}).$$

Hence, $v_{h(m)}(\hat{z}_{h(m)}, \theta_{h(m)}^\mathcal{J}) = v_{h(m)}(z_{h(m)}, \theta_{h(m)}^\mathcal{J}) = \pi_{h(m)}(\theta^\mathcal{J})$, for all $1 \leq m < n$. It follows that $\hat{z}$ belongs to $\mathbf{Z}_{h(n-1)}(\theta^\mathcal{J})$, which is the constraint set of $h(n)$'s optimization problem. Immediately, $v_{h(n)}(\hat{z}_{h(n)}, \theta_{h(n)}^\mathcal{J}) \leq \pi_{h(n)}(\theta^\mathcal{J}) = v_{h(n)}(z_{h(n)}, \theta_{h(n)}^\mathcal{J})$, a contradiction. $\square$

In obtaining the efficiency of the jungle equilibria in $\mathbf{Z}^*(\theta^\mathcal{J}, h)$, we do not require any monotonicity assumption on preferences for commodities. Without monotonicity, an individual in the jungle may be better off disposing of one (or more) goods. The exact clearing condition in [Definition 1\(b\)](#) avoids this. Feasibility could be weakened to the following: a trade profile z is “feasible” at $\theta^\mathcal{J}$ if $z_j \in Z_j(\theta_j^\mathcal{J})$ for all $j \in \mathcal{J}$ and $\sum_{j \in \mathcal{J}} z_j \leq \mathbf{0}$. This weakened feasible set is still nonempty and compact, so [Proposition 1](#) and [Proposition 2](#) follow verbatim, without any monotonicity assumption. Monotonicity is relevant only if one wants the weak inequality to bind, a *no-waste* reading in which disposal is never desirable or imposes additional costs on individuals or on society. Monotonicity alone does not deliver this: the set of feasible trades $Z_j(\theta_j^\mathcal{J})$ is bounded, so an individual who strictly prefers more of a disposed good ℓ may have no room to absorb this.

The local jungle equilibria in $\mathbf{Z}^*(\theta^\mathcal{J}, h)$ are efficient, and thus undominated; we focus on them for the remainder of the paper. In [Appendix A](#) we show that, under an additional assumption, which we call hierarchical improvement, any equilibrium in $\mathbf{Z}^*(\theta^\mathcal{J}, h)$ weakly payoff dominates every other local equilibrium in the jungle. In their setting without production, [Piccione and Rubinstein \(2007\)](#) obtain a condition of this kind from primitives. They call a jungle *smooth* when each utility function is strictly quasi-concave, continuously differentiable, with strictly positive gradient vector, and each consumption set has the form $\{x : g^i(x) \leq 0\}$ with g^i quasi-convex and differentiable, with $\nabla g^i \gg \mathbf{0}$. In a smooth jungle, whenever an individual can improve upon a feasible allocation while keeping stronger individuals' bundles fixed, she can do so by taking unallocated goods or goods from a single weaker individual, and the jungle equilibrium is unique. Their argument does not carry over to our setting, for two reasons. First, it relies on strict monotonicity of preferences, which we do not impose. Second, their notion of feasibility allows free disposal: the victim remains feasible after surrendering what the deviant takes, and whatever the deviant releases can simply be discarded. Under the exact clearing of [Definition 1\(b\)](#), by contrast, a bilateral deviation requires the victim to absorb what the deviant releases. More generally, any released commodities must be accommodated through a feasible reshuffle that preserves the payoffs of third parties. [Appendix A](#) also shows that, under hierarchical improvement, strengthening the concavity of $v_i(\cdot, \theta_i^\mathcal{J})$ to strict concavity delivers a unique local jungle equilibrium.

3.2. Random Hierarchies and Occupational Equilibria

Our concept of local jungle equilibrium has been stated for *fixed* occupational profile $\theta^\mathcal{J} \in \Theta^\mathcal{J}$ and power relation $h \in \mathcal{H}(\mathcal{J})$. Any equilibrium trade profile $z^* \in \mathbf{Z}^*(\theta^\mathcal{J}, h)$ generates the same payoff to individual $h(n) \in \mathcal{J}$, namely $\pi_{h(n)}(\theta^\mathcal{J})$. A main goal in our work is to consider the coexistence of the market and the jungle. To investigate this, we study occupational choice

and outline a further extension to participation decisions. First, we let individuals in the jungle decide which occupations to choose; i.e., each individual $i \in \mathcal{J}$ will optimally choose an occupation in $\Theta_i^{\mathcal{J}}$, the set of occupations available for her in the jungle. (Later, we do the same for the market economy.) A further extension would let individuals decide whether to join the jungle or the market in anticipation of the resulting occupational choices. A stable situation for our society, i.e., a *social equilibrium*, would endogenize the size and composition of these two economies.

Here we focus on the first extension and consider how those individuals who are already in the jungle choose occupations optimally. Since payoffs in a local jungle equilibrium depend on the hierarchy, it is entirely plausible that individuals would invest in achieving higher rankings in the jungle hierarchy. Skaperdas (1992) studies a static model in which individuals allocate limited resources between the cooperative production of a consumption good and “arms.” We do not consider individuals fighting their way to the top of the hierarchy in the jungle, taking instead the power relation as random.⁴ Random hierarchies capture uncertainty about the exercise of coercive power. This interpretation is consistent with research on conflicts within authoritarian ruling coalitions (Svolik, 2009; Acemoglu et al., 2008) and on stochastic changes in the distribution of political power (Acemoglu et al., 2015). We represent this uncertainty by assuming that occupational choices precede the realization of the hierarchy governing allocation.

Still working with a fixed jungle $\mathcal{J} \neq \emptyset$, recall the set of hierarchies $\mathcal{H}(\mathcal{J})$ contains all bijections $h: \{1, 2, \dots, |\mathcal{J}|\} \rightarrow \mathcal{J}$. Since the jungle $\mathcal{J}$ is finite, there are finitely many bijections. Impose a uniform distribution on $\mathcal{H}(\mathcal{J})$, so that each power relation is chosen with probability $1/|\mathcal{J}|!$. In our framework, individuals anticipate that joining the jungle will give them an equal chance to occupy a certain position in the jungle hierarchy. For most of our results, this is a normalization; we only require that individuals not choose their positions in the hierarchy themselves: any fixed distribution on $\mathcal{H}(\mathcal{J})$ will do. What is important is that the ranking in the jungle hierarchy is not known when occupations are chosen, and does not depend on the resources an individual brings to the jungle. Uniformity of chances just adds symmetry.

Once individuals in the jungle choose occupations and $\theta^{\mathcal{J}} \in \Theta^{\mathcal{J}}$ is fixed, each $i \in \mathcal{J}$ obtains in expectation a *jungle payoff* equal to

$$\bar{\pi}_i(\theta^{\mathcal{J}}) = \mathbb{E}_{\mathcal{H}(\mathcal{J})} [\pi_{h^{-1}(i)}(\theta^{\mathcal{J}})] = \frac{1}{|\mathcal{J}|!} \sum_{h \in \mathcal{H}(\mathcal{J})} \pi_{h^{-1}(i)}(\theta^{\mathcal{J}}). \quad (2)$$

All individuals in the jungle economy anticipate that, given occupational profile $\theta^{\mathcal{J}}$ and hierarchy h , a local jungle equilibrium in $\mathbf{Z}^*(\theta^{\mathcal{J}}, h)$ is selected. Every equilibrium in this family provides individual i with the payoff $\pi_{h^{-1}(i)}(\theta^{\mathcal{J}})$ when the prevailing hierarchy is $h \in \mathcal{H}(\mathcal{J})$.⁵ This is the expression inside the expectations operator in Equation 2. Averaging over all possible hierarchies (under the uniform distribution) obtains i 's jungle payoff $\bar{\pi}_i(\theta^{\mathcal{J}})$.

An occupational equilibrium in the jungle economy is a profile of occupational choices that

⁴Cortés-McPherson (2019) recounts a case from the small-scale gold mining area of Madre de Dios, Peru: Gregoria Casas Huamanhuillca, “la Reina del Oro”, a Quechua-speaking woman from the Peruvian highlands who became a key player in the region.

⁵We slightly abuse notation here and write $\pi_{h^{-1}(i)}(\theta^{\mathcal{J}})$ for the equilibrium payoff of individual i at the given occupational profile, when her ranking under the hierarchy h is $h^{-1}(i)$.

gives no individual incentives to deviate unilaterally.

Definition 4. An occupational equilibrium in the jungle $\mathcal{J}$ is a profile $\theta^{\mathcal{J}*} = (\theta_i^{\mathcal{J}*}) \in \Theta^{\mathcal{J}}$ such that, for each individual $i \in \mathcal{J}$ and each alternative occupation $\theta_i^{\mathcal{J}} \in \Theta_i^{\mathcal{J}}$,

$$\bar{\pi}_i(\theta_i^{\mathcal{J}*}, \theta_{-i}^{\mathcal{J}*}) \geq \bar{\pi}_i(\theta_i^{\mathcal{J}}, \theta_{-i}^{\mathcal{J}*}). \quad (3)$$

3.3. Existence of Occupational Equilibria in the Jungle

An occupational equilibrium in the jungle is a (pure) Nash equilibrium of the game in which individuals simultaneously choose occupations and anticipate the expected payoffs from the jungle allocation. Even when the occupation sets are compact subsets of Euclidean spaces, the expected payoff function in Equation 2 need not be continuous, so neither Berge's Maximum Theorem nor standard fixed point arguments apply to the occupational game in the jungle, as it stands. Hence, we cannot take existence of an occupational equilibrium for granted.

There are two issues in proving existence of occupational equilibria in the jungle. The first, common to parametric optimization problems, is that individual i 's occupational choice affects not only her utility function but also the set of individually feasible trade positions she has at her disposal in the jungle economy. To deal with this, we exploit the structure given by Assumption 1 to the set of occupational choices $\Theta_i^{\mathcal{J}}$, the correspondence $Z_i: \Theta_i^{\mathcal{J}} \rightrightarrows \mathbb{R}^L$, and the valuation function v_i .

The second issue is novel and comes from the serial recursion used to obtain existence of local jungle equilibria in Proposition 1. In this recursion, the constraint set at each stage, except for the initial one, is the solution set of the preceding stage. Thus, when treated as a correspondence, the constraint correspondence of all but the initial step in our nested optimization problem is endogenous, and cannot simply be assumed to be a continuous correspondence. As a result, this nesting need not deliver continuity of the individual value functions. The next examples illustrate this issue and point towards ways to deal with it.

Example 1. There is only one commodity in this economy, so $L = 1$. The jungle is populated by two individuals, $\mathcal{J} = \{1, 2\}$, with $\Theta_1^{\mathcal{J}} = [0, 1]$ and $\Theta_2^{\mathcal{J}} = \{1\}$, so only the first individual has occupational choices. For both $i = 1, 2$, let $Z_i(\theta_i^{\mathcal{J}}) = [-1, 1]$ and $v_i(z, \theta_i^{\mathcal{J}}) = \theta_i^{\mathcal{J}} z$ on this interval, with value $-\infty$ outside it. Clearly, all of the conditions in Assumption 1 hold in this setting.

For simplicity, express the set of feasible trade profiles in this jungle economy as the line segment $\mathbf{Z} = \{t : t \in [-1, 1]\}$, with the understanding that $t \in \mathbf{Z}$ allocates quantity t to individual 1 and quantity $-t$ to individual 2; we omit dependence of $\mathbf{Z}$ on the occupational profile since each $Z_i(\theta_i^{\mathcal{J}})$ does not depend on $\theta_i^{\mathcal{J}}$.

Consider the hierarchy $h \in \mathcal{H}(\mathcal{J})$ that sets $h(1) = 1$. Clearly, the first dictator chooses $t = 1$ if $\theta_{h(1)}^{\mathcal{J}} > 0$, but at the occupational choice $\theta_{h(1)}^{\mathcal{J}} = 0$, she is indifferent over the entire $\mathbf{Z}$. Thus, $h(1)$'s solution correspondence in her optimization problem, which is the feasible correspondence in $h(2)$'s optimization problem, is

$$\mathbf{Z}_{h(1)}(\theta^{\mathcal{J}}) = \begin{cases} [-1, 1] & : \theta_{h(1)}^{\mathcal{J}} = 0, \\ \{1\} & : 0 < \theta_{h(1)}^{\mathcal{J}} \leq 1. \end{cases}$$

This correspondence is upper hemicontinuous but not lower hemicontinuous. Readily,

$$\pi_{h(1)}(\boldsymbol{\theta}^{\mathcal{J}}) = \theta_{h(1)}^{\mathcal{J}} \quad \text{and} \quad \pi_{h(2)}(\boldsymbol{\theta}^{\mathcal{J}}) = \begin{cases} 1 & : \theta_{h(1)}^{\mathcal{J}} = 0, \\ -1 & : 0 < \theta_{h(1)}^{\mathcal{J}} \leq 1. \end{cases}$$

Thus, with $h(1) = 1$ at the top of the hierarchy, individual 1's payoff function coming from local jungle equilibria is continuous, but individual 2's payoff function exhibits a jump at $\theta_1^{\mathcal{J}} = 0$. Under the reverse order, individual 2 chooses $t = -1$ and obtains 1, and individual 1 obtains $-\theta_1^{\mathcal{J}}$. But the averaging over the two power relations does not repair the discontinuity in the expected payoff function of the second individual: $\bar{\pi}_2(\boldsymbol{\theta}^{\mathcal{J}}) = 1$ for $\theta_1^{\mathcal{J}} = 0$ and $\bar{\pi}_2(\boldsymbol{\theta}^{\mathcal{J}}) = 0$ for all $\theta_1^{\mathcal{J}} > 0$. $\diamond$

The discontinuity in the jungle payoff function of individual 2 in [Example 1](#) is generated by the multiplicity of solutions at one of individual 1's occupational choices. Replacing concavity of the valuation function with strict concavity makes the optimizing individual's trade unique at each stage, but need not determine the other individuals' trades. Because of this latter point, it is not sufficient to guarantee the continuity of payoffs. The next example illustrates this new difficulty. All individual valuations are strictly concave, and the most powerful individual's optimal trade is unique and does not change with occupational choices. Nevertheless, a weaker individual's jungle payoff turns out to be discontinuous.

Example 2. There are two goods in this economy, so $L = 2$. The jungle is now populated by three individuals, $\mathcal{J} = \{1, 2, 3\}$, with $\Theta_1^{\mathcal{J}} = \Theta_2^{\mathcal{J}} = \{1\}$ and $\Theta_3^{\mathcal{J}} = [0, 1]$. Thus, only the third individual has occupational choices. Writing $z_i = (x_i, y_i)$ for an individual trade, let

$$\begin{aligned} Z_1(1) &= [-1, 1]^2, & Z_2(1) &= [-1, 0] \times [0, 1], & \text{and} \\ Z_3(\theta_3^{\mathcal{J}}) &= \{(x_3, y_3) \in [-1, 0] \times [0, 1] : \theta_3^{\mathcal{J}} x_3 + y_3 \geq 0\}. \end{aligned}$$

Individual 2 can supply up to one unit of the first good without using the second good as an input. Individual 3's occupation determines her input requirement: supplying $-x$ units of the first good requires at least $-\theta_3^{\mathcal{J}} x$ units of the second. On the respective individual feasible sets, let valuations be defined by

$$\begin{aligned} v_1((x_1, y_1), 1) &= 2x_1 - x_1^2 - y_1^2, \\ v_i((x_i, y_i), \theta_i^{\mathcal{J}}) &= -x_i^2 - y_i^2, \quad \text{for } i = 2, 3, \end{aligned}$$

with value $-\infty$ outside those sets. Notice that individual 1 prefers to receive one unit of the first good and not to trade the second. Individuals 2 and 3 bear costs when required to produce.

All conditions in [Assumption 1](#) are present. In particular, the correspondence Z_3 is continuous. Moreover, since zero belongs to both $Z_2(1)$ and $Z_3(\theta_3^{\mathcal{J}})$, their aggregate with $Z_1(1)$ contains $[-1, 1]^2$, and thus $\mathbf{0}$ is in its interior. Notice the valuation functions are not only continuous on their feasible graphs but also strictly concave in trade. The individual feasible sets, however, are not strictly convex.

Consider the hierarchy with $h(1) = 1$, $h(2) = 2$ and $h(3) = 3$. The strongest individual's

unique optimal trade is $(1,0)$. This trade is feasible at every occupational profile. Thus individual $h(1) = 1$ always chooses $(1,0)$ and obtains payoff 1. Because the second coordinates of the remaining individuals' trades are nonnegative, the clearing condition now requires $y_2 = y_3 = 0$. If $\theta_3^{\mathcal{J}} > 0$, individual 3's input requirement forces $x_3 = 0$. In this case, individual 2 must supply the entire unit of the first commodity consumed by the first dictator. If instead $\theta_3^{\mathcal{J}} = 0$, either remaining individual can supply it. The first dictator's solution correspondence is therefore

$$\mathbf{Z}_{h(1)}(\boldsymbol{\theta}^{\mathcal{J}}) = \begin{cases} \{((1,0), (-t,0), (t-1,0)) : t \in [0,1]\}, & \theta_3^{\mathcal{J}} = 0, \\ \{((1,0), (-1,0), (0,0))\}, & 0 < \theta_3^{\mathcal{J}} \leq 1. \end{cases}$$

This correspondence is upper hemicontinuous but not lower hemicontinuous at $\theta_3^{\mathcal{J}} = 0$. At this occupation, individual $h(2) = 2$ chooses $t = 0$ and leaves all production to individual $h(3) = 3$. Consequently,

$$\pi_{h(2)}(\boldsymbol{\theta}^{\mathcal{J}}) = \begin{cases} 0, & \theta_3^{\mathcal{J}} = 0, \\ -1, & 0 < \theta_3^{\mathcal{J}} \leq 1. \end{cases}$$

Averaging over hierarchies does not repair the discontinuity in the payoff for individual 2. In the power orders $(1,3,2)$ and $(3,1,2)$, listed from strongest to weakest, individual 3 chooses $\mathbf{0}$ and individual 1 takes $(1,0)$, leaving individual 2 with $(-1,0)$ and a payoff of -1 . In the orders $(2,1,3)$, $(2,3,1)$ and $(3,2,1)$, individual 2 chooses $\mathbf{0}$ and obtains payoff 0. These five payoffs do not depend on $\theta_3^{\mathcal{J}}$, and hence $\bar{\pi}_2(\boldsymbol{\theta}^{\mathcal{J}}) = -1/3$ at $\theta_3^{\mathcal{J}} = 0$ and $\bar{\pi}_2(\boldsymbol{\theta}^{\mathcal{J}}) = -1/2$ for all $0 < \theta_3^{\mathcal{J}} \leq 1$. $\diamond$

To spell out the issue illustrated by these examples, fix a hierarchy h in the jungle $\mathcal{J}$. For each position $n = 1, 2, \dots, |\mathcal{J}|$, the correspondence $\mathbf{Z}_{h(n)} : \boldsymbol{\Theta}^{\mathcal{J}} \rightrightarrows \mathbb{R}^{|\mathcal{J}| \times L}$ employed in the proof of [Proposition 1](#) can be expressed as

$$\mathbf{Z}_{h(n)}(\boldsymbol{\theta}^{\mathcal{J}}) = \left\{ \mathbf{z} \in \mathbf{Z}_{h(n-1)}(\boldsymbol{\theta}^{\mathcal{J}}) : v_{h(n)}(z_{h(n)}, \theta_{h(n)}^{\mathcal{J}}) = \pi_{h(n)}(\boldsymbol{\theta}^{\mathcal{J}}) \right\}. \quad (4)$$

Continuity of $\pi_{h(n)}(\boldsymbol{\theta}^{\mathcal{J}})$ on $\boldsymbol{\Theta}^{\mathcal{J}}$ for every hierarchy h would imply continuity of the expected payoff function in [Equation 2](#), since it is an averaging over finitely many power orders. But $\pi_{h(n)}$ is the value function coming from individual $h(n)$'s optimization problem, so one is required to show, not assume, its continuity. Moreover, $\mathbf{Z}_{h(n-1)}$ is the constraint correspondence at stage n in the recursion but the solution correspondence of the previous stage, and thus it need not be lower hemicontinuous even when it is upper hemicontinuous.

[Example 1](#) illustrates the problems created when a stage in the nested optimization problem of our serial dictatorship procedure has multiple solutions. The difficulty of [Example 2](#) has a different source: it relates to how weaker individuals can accommodate a stronger individual's chosen trade, even when that optimal trade is unique. To deal with these difficulties, we strengthen the structure of the jungle economy by imposing strict concavity of valuations on trade and strict convexity of individual feasible sets. We next show that these stronger conditions are sufficient to guarantee that the nested optimization problem in [Proposition 1](#) has, at each stage, a constraint correspondence that is not only upper hemicontinuous but also

lower hemicontinuous.

Proposition 3. Suppose that, complementing the conditions provided in [Assumption 1](#) as applied to the jungle, for all $i \in \mathcal{J}$ the correspondence $Z_i: \Theta_i^\mathcal{J} \rightrightarrows \mathbb{R}^L$ has strictly convex values⁶ in $\mathbb{R}^L$ and the valuation function $v_i(\cdot, \theta_i^\mathcal{J})$ is strictly concave on $Z_i(\theta_i^\mathcal{J})$, for all $\theta_i^\mathcal{J} \in \Theta_i^\mathcal{J}$. Given any hierarchy $h \in \mathcal{H}(\mathcal{J})$, the following hold:

- (a) For every $n = 0, 1, \dots, |\mathcal{J}|$, the correspondence $\theta^\mathcal{J} \rightrightarrows \mathbf{Z}_{h(n)}(\theta^\mathcal{J})$ defined in [Equation 4](#) is continuous and has nonempty, compact and convex values.
- (b) For every $n = 1, \dots, |\mathcal{J}|$, the payoff function $\theta^\mathcal{J} \mapsto \pi_{h(n)}(\theta^\mathcal{J})$ is continuous on $\Theta^\mathcal{J}$.

Proof. Fix a hierarchy $h \in \mathcal{H}(\mathcal{J})$ for the rest of the proof. By [Lemma 1](#) and [Proposition 1](#), for any occupational profile $\theta^\mathcal{J}$, the sets $\mathbf{Z}_{h(n)}(\theta^\mathcal{J})$ are nonempty and compact at every stage $n = 0, 1, \dots, |\mathcal{J}|$. Their convexity follows from the argument in [Corollary 1\(a\)](#).

For $n \geq 1$, strict concavity of the valuation function implies that any two trade profiles in $\mathbf{Z}_{h(n)}(\theta^\mathcal{J})$ agree in the individual trade positions of $h(1), h(2), \dots, h(n)$. Indeed, if two maximizers at stage n in the recursion differed in individual $h(n)$'s position, their midpoint would belong to the convex set $\mathbf{Z}_{h(n-1)}(\theta^\mathcal{J})$ and give this individual strictly more than $\pi_{h(n)}(\theta^\mathcal{J})$, which is impossible. The same argument at the preceding stages, together with the nesting of the constraint sets in the proof of [Proposition 1](#), gives agreement in all earlier positions.

We claim that, at each stage, the upper hemicontinuity of the correspondence $\mathbf{Z}_{h(n)}$ implies its lower hemicontinuity. To see this, fix some $n \in \{0, 1, \dots, |\mathcal{J}|\}$ and suppose that $\mathbf{Z}_{h(n)}$ is upper hemicontinuous at occupational profile $\theta^\mathcal{J}$. Consider any sequence $\theta^r \rightarrow \theta^\mathcal{J}$ in $\Theta^\mathcal{J}$ and any target $z \in \mathbf{Z}_{h(n)}(\theta^\mathcal{J})$. For each $r = 1, 2, \dots$, choose $z^r \in \mathbf{Z}_{h(n)}(\theta^r)$ with minimum distance to z . Such trade profiles exist by the nonemptiness and compactness of the sets $\mathbf{Z}_{h(n)}(\theta^r)$.

From [Assumption 1\(b\)](#), the individual feasible correspondences are locally bounded. Thus, if the sequence $\{z^r\}$ does not converge to z , the upper hemicontinuity of $\mathbf{Z}_{h(n)}$ at $\theta^\mathcal{J}$ gives a subsequence, which we still index by r , such that $z^r \rightarrow \hat{z} \in \mathbf{Z}_{h(n)}(\theta^\mathcal{J})$ and $\hat{z} \neq z$. Along this subsequence, define

$$\tilde{z}^r = z^r + \frac{1}{2}(z - \hat{z}).$$

If $z_i = \hat{z}_i$ for $i \in \mathcal{J}$, then $\tilde{z}_i^r = z_i^r \in Z_i(\theta_i^r)$. If instead $z_i \neq \hat{z}_i$, strict convexity of the individual trading sets implies

$$\tilde{z}_i^r \rightarrow \frac{z_i + \hat{z}_i}{2} \in \text{int } Z_i(\theta_i^\mathcal{J}).$$

The continuity and convexity of the correspondence Z_i on $\Theta_i^\mathcal{J}$ imply that a fixed neighborhood of this midpoint is contained in $Z_i(\theta_i^r)$, for all sufficiently large r . To see this, choose a simplex contained in $Z_i(\theta_i^\mathcal{J})$ with the midpoint in its interior. Lower hemicontinuity allows the vertices of this simplex to be approximated by points in $Z_i(\theta_i^r)$. By convexity, their convex hulls are contained in $Z_i(\theta_i^r)$ and still contain a fixed neighborhood of the midpoint. Consequently, $\tilde{z}_i^r \in Z_i(\theta_i^r)$ for all sufficiently large r .

⁶That is, for any two distinct $z_i, z_i' \in Z_i(\theta_i^\mathcal{J})$, for any $\lambda \in (0, 1)$, the trade position $\lambda z_i + (1 - \lambda)z_i' \in \text{int}_{\mathbb{R}^L} Z_i(\theta_i^\mathcal{J})$. This allows $Z_i(\theta_i^\mathcal{J})$ to be a singleton, e.g., $Z_i(\theta_i^\mathcal{J}) = \{0\}$, but excludes non-degenerate, lower dimensional individual trade sets.

Since there are finitely many individuals, all trading positions in $\tilde{z}^r$ are individually feasible, again for sufficiently large r . Moreover,

$$\sum_{i \in \mathcal{J}} \tilde{z}_i^r = \sum_{i \in \mathcal{J}} z_i^r + \frac{1}{2} \left(\sum_{i \in \mathcal{J}} z_i - \sum_{i \in \mathcal{J}} \hat{z}_i \right) = \mathbf{0}.$$

Hence $\tilde{z}^r \in \mathbf{Z}(\theta^r)$. For $n \geq 1$, the agreement of z and $\hat{z}$ in the first n positions in the hierarchy also shows that

$$\tilde{z}_{h(m)}^r = z_{h(m)}^r, \quad \text{for all } m = 1, \dots, n.$$

These equalities imply that $\tilde{z}^r$ attains the same payoff as z^r at each of the first n nested optimization problems. Thus $\tilde{z}^r \in \mathbf{Z}_{h(n)}(\theta^r)$ for all sufficiently large r . Note this also holds when $n = 0$. Now, the choice of z^r as a point of minimum distance to z gives

$$\|z^r - z\| \leq \|\tilde{z}^r - z\|.$$

Taking limits along the subsequence obtains

$$\|\hat{z} - z\| \leq \frac{1}{2} \|\hat{z} - z\|,$$

which contradicts $\hat{z} \neq z$. Therefore $z^r \rightarrow z$, which shows that the lower hemicontinuity of $\mathbf{Z}_{h(n)}$ at θ^J follows from its upper hemicontinuity at θ^J .

We are ready to establish the continuity properties of (a) and (b) in the proposition.

Initial Step. The individual feasible correspondences are locally bounded and have closed graphs. The feasibility constraint set $F = \{z : \sum_{i \in \mathcal{J}} z_i = \mathbf{0}\}$ is also closed. Therefore, the correspondence $\mathbf{Z}_{h(0)} = \mathbf{Z}$ defined on Θ^J is upper hemicontinuous. The preceding argument gives lower hemicontinuity, so $\mathbf{Z}_{h(0)}$ is continuous.

Induction Step. For $1 \leq n \leq |\mathcal{J}|$, suppose that $\mathbf{Z}_{h(n-1)}$ is a continuous correspondence. Individual $h(n)$'s optimization problem can be expressed as

$$\pi_{h(n)}(\theta^J) = \max_z \left\{ v_{h(n)}(z_{h(n)}, \theta_{h(n)}^J) : z \in \mathbf{Z}_{h(n-1)}(\theta^J) \right\}.$$

From [Assumption 1\(c\)](#), the objective function is finite and continuous on the graph of the constraint correspondence. Berge's Theorem of the Maximum implies that $\pi_{h(n)}$ is continuous and that $\mathbf{Z}_{h(n)}$ is upper hemicontinuous. The preceding argument gives lower hemicontinuity, hence continuity of $\mathbf{Z}_{h(n)}$.

This establishes part (a) for $n = 0, 1, \dots, |\mathcal{J}|$ and part (b) for $n = 1, \dots, |\mathcal{J}|$. $\square$

We can now state our existence result for the jungle economy. In the main body of the paper we focus on pure occupational equilibria. This will allow us to compare in a relatively simple way properties of occupational equilibria in the jungle and in the market. For that purpose, we assume that any convex combination of best replies is also a best reply. In [Appendix B](#), we consider existence of mixed occupational equilibria, which do not rely on any additional assumption.

Theorem 1. *Let the conditions of Proposition 3 be in place. Suppose in addition that, for every individual $i \in \mathcal{J}$ and every $\theta_{-i}^\mathcal{J} \in \times_{j \neq i} \Theta_j^\mathcal{J}$, the set of maximizers of $\theta_i^\mathcal{J} \mapsto \bar{\pi}_i(\theta_i^\mathcal{J}, \theta_{-i}^\mathcal{J})$ over $\Theta_i^\mathcal{J}$ is convex. Then an occupational equilibrium in the jungle, in the sense of Definition 4, exists.*

Proof. We know, from Proposition 3(b), that each individual's payoff function is continuous in occupational profiles for every fixed hierarchy h . Since $\mathcal{H}(\mathcal{J})$ is finite, the expected payoff $\bar{\pi}_i$ is continuous on $\Theta^\mathcal{J}$, for every $i \in \mathcal{J}$.

From Assumption 1(a), as imposed on the jungle, $\Theta^\mathcal{J} = \times_{i \in \mathcal{J}} \Theta_i^\mathcal{J}$ is a nonempty, compact and convex subset of a finite-dimensional normed vector space. For each individual i , define the best reply correspondence

$$BR_i(\theta_{-i}^\mathcal{J}) \equiv \operatorname{argmax} \{ \bar{\pi}_i(\theta_i^\mathcal{J}, \theta_{-i}^\mathcal{J}) : \theta_i^\mathcal{J} \in \Theta_i^\mathcal{J} \}.$$

Berge's Theorem of the Maximum implies that BR_i is upper hemicontinuous and has nonempty, compact values. The additional assumption in the theorem makes these values convex. Hence the product correspondence

$$BR(\theta^\mathcal{J}) \equiv \times_{i \in \mathcal{J}} BR_i(\theta_{-i}^\mathcal{J})$$

maps $\Theta^\mathcal{J}$ into itself, is upper hemicontinuous, and has nonempty, compact and convex values. Applying Kakutani's Fixed Point Theorem obtains an occupational profile $\theta^{\mathcal{J}*} \in \Theta^\mathcal{J}$ such that $\theta^{\mathcal{J}*} \in BR(\theta^{\mathcal{J}*})$. Thus, for every $i \in \mathcal{J}$ and every alternative occupation $\theta_i^\mathcal{J} \in \Theta_i^\mathcal{J}$, we have

$$\bar{\pi}_i(\theta_i^{\mathcal{J}*}, \theta_{-i}^{\mathcal{J}*}) \geq \bar{\pi}_i(\theta_i^\mathcal{J}, \theta_{-i}^{\mathcal{J}*}). \quad \square$$

The existence argument in Theorem 1 follows the standard best reply fixed-point approach of Debreu (1952), Fan (1952) and Glicksberg (1952). Our Proposition 3 establishes continuity of the expected occupational payoffs generated by the serial dictatorship in the jungle. Given this continuity, compact and convex occupational menus and convex sets of occupational best replies permit an application of Kakutani's Fixed Point Theorem. The extra assumption for our existence of a pure occupational equilibrium in the jungle in Theorem 1 restricts an endogenous object. The following special case provides a sufficient condition on primitives.

Corollary 2. *Let the conditions of Proposition 3 be in place. Assume that, for every individual $i \in \mathcal{J}$ in the jungle, the feasible trade set $Z_i(\theta_i^\mathcal{J})$ is independent of $\theta_i^\mathcal{J}$, and denote it simply by Z_i . If the function $(z_i, \theta_i^\mathcal{J}) \mapsto v_i(z_i, \theta_i^\mathcal{J})$ is jointly concave on $Z_i \times \Theta_i^\mathcal{J}$, for $i \in \mathcal{J}$, then a pure occupational equilibrium in the jungle exists.*

Proof. Fix an individual $i \in \mathcal{J}$ and a hierarchy h , and let n be the position for which $h(n) = i$. Fix also an occupational profile $\theta_{-i}^\mathcal{J}$ for the other individuals. Because the individual feasible trade sets are independent of occupations, changing $\theta_i^\mathcal{J}$ does not change the initial feasible set $Z = Z_{h(0)}$. It also does not change the objective functions of the individuals preceding i in the hierarchy. Applying the recursion in Proposition 1 successively, the constraint set $Z_{h(n-1)}(\theta_i^\mathcal{J}, \theta_{-i}^\mathcal{J})$ is therefore independent of $\theta_i^\mathcal{J}$. Moreover, this set is nonempty, compact and convex.

Consider two occupations $\theta_i^\mathcal{J}, \hat{\theta}_i^\mathcal{J} \in \Theta_i^\mathcal{J}$, and choose optimal trade profiles

$$z \in Z_{h(n)}(\theta_i^\mathcal{J}, \theta_{-i}^\mathcal{J}) \quad \text{and} \quad \hat{z} \in Z_{h(n)}(\hat{\theta}_i^\mathcal{J}, \theta_{-i}^\mathcal{J}).$$

By our previous observation, both $\mathbf{z}$ and $\hat{\mathbf{z}}$ belong to the same convex constraint set. For every $\lambda \in [0, 1]$, their convex combination $\lambda \mathbf{z} + (1 - \lambda) \hat{\mathbf{z}}$ is consequently feasible in individual $h(n) = i$'s optimization problem at occupation $\lambda \theta_i^\mathcal{J} + (1 - \lambda) \hat{\theta}_i^\mathcal{J}$. Joint concavity of the valuation function now implies

$$\begin{aligned} \pi_{h(n)}(\lambda \theta_i^\mathcal{J} + (1 - \lambda) \hat{\theta}_i^\mathcal{J}, \boldsymbol{\theta}_{-i}^\mathcal{J}) &\geq v_i(\lambda \mathbf{z}_i + (1 - \lambda) \hat{\mathbf{z}}_i, \lambda \theta_i^\mathcal{J} + (1 - \lambda) \hat{\theta}_i^\mathcal{J}) \\ &\geq \lambda v_i(\mathbf{z}_i, \theta_i^\mathcal{J}) + (1 - \lambda) v_i(\hat{\mathbf{z}}_i, \hat{\theta}_i^\mathcal{J}) \\ &= \lambda \pi_{h(n)}(\theta_i^\mathcal{J}, \boldsymbol{\theta}_{-i}^\mathcal{J}) + (1 - \lambda) \pi_{h(n)}(\hat{\theta}_i^\mathcal{J}, \boldsymbol{\theta}_{-i}^\mathcal{J}). \end{aligned}$$

Thus individual i 's payoff under the hierarchy h is concave in her own occupation. This holds for every hierarchy in $\mathcal{H}(\mathcal{J})$, and thus averaging over power relations preserves the inequality. We conclude that the expected payoff $\bar{\pi}_i(\cdot, \boldsymbol{\theta}_{-i}^\mathcal{J})$ is concave, and thus its set of maximizers is convex. [Theorem 1](#) thus applies. $\square$

4. The Market

A major difference between the market and the jungle is in the enforcement of contracts and property rights. In the market, impersonal transactions are enforced by an anonymous, centralized institution that, in addition, organizes the supply of the numeraire. So, in the market the numeraire is tradable, and individual choices are settled by posted prices.

4.1. Local and Occupational Equilibria in the Market

Our model of a market economy follows [Makowski and Ostroy \(1995\)](#); see related analysis in [Makowski and Ostroy \(1987\)](#). We continue to impose [Assumption 1](#), adapting it to the market economy, and we work with a nonempty market throughout this section.

Definition 5. Fix an occupational profile $\boldsymbol{\theta}^\mathcal{M} = (\theta_i^\mathcal{M}) \in \boldsymbol{\Theta}^\mathcal{M}$ for the market economy $\mathcal{M}$. A pair $(\mathbf{z}, p) \in \mathbf{Z}(\boldsymbol{\theta}^\mathcal{M}) \times \mathbb{R}^L$ is a market equilibrium at $\boldsymbol{\theta}^\mathcal{M}$ if, for each individual $i \in \mathcal{M}$ and every $\mathbf{z}'_i \in \mathbb{R}^L$,

$$v_i(\mathbf{z}_i, \theta_i^\mathcal{M}) - p \cdot \mathbf{z}_i \geq v_i(\mathbf{z}'_i, \theta_i^\mathcal{M}) - p \cdot \mathbf{z}'_i.$$

The above inequality is i 's optimality condition under quasilinear preferences with the price of the numeraire normalized to one: she chooses a trade and a monetary holding to maximize $v_i(\mathbf{z}'_i, \theta_i^\mathcal{M}) + m'_i$ subject to the trading budget constraint $p \cdot \mathbf{z}'_i + m'_i = 0$. The market clearing condition for the L commodities is embedded in the requirement that $\mathbf{z} \in \mathbf{Z}(\boldsymbol{\theta}^\mathcal{M})$. In equilibrium, the market for the numeraire also clears. Notice we do not need to impose the restriction $\mathbf{z}'_i \in Z_i(\theta_i^\mathcal{M})$, since $v_i(\mathbf{z}'_i, \theta_i^\mathcal{M}) = -\infty$ for any $\mathbf{z}'_i$ not in $Z_i(\theta_i^\mathcal{M})$. We allow prices of either sign: without monotonicity of preferences a commodity may be a nuisance, and the restriction $p \geq \mathbf{0}$ would be an additional hypothesis rather than a conclusion. But this does not play any role in our analysis.

We sometimes refer to a market equilibrium at $\boldsymbol{\theta}^\mathcal{M}$ as a *local market equilibrium*. [Assumption 1\(d\)](#), as imposed on the market economy, implies that all commodity markets are open in

the sense of [Makowski and Ostroy \(1995\)](#): every commodity $\ell = 1, \dots, L$ can be potentially supplied at every occupational profile.⁷ Thus, in our market economy occupational choices can affect trading possibilities and equilibrium prices, and markets are complete. Note however that this does not require every commodity to be traded in equilibrium.

The next proposition establishes the existence of a local market equilibrium. Our proof uses a standard supporting-price argument; we include it for completeness.⁸ [Assumption 1\(d\)](#) ensures that, in any local market equilibrium, the price of the numeraire is strictly positive, so our price normalization implicit in [Definition 5](#) is without loss of generality.

Proposition 4. *For any occupational profile $\theta^M \in \Theta^M$, a local market equilibrium exists.*

Proof. Fix an occupational profile $\theta^M \in \Theta^M$. For each $\bar{z} \in \sum_i Z_i(\theta_i^M)$, define

$$\hat{g}(\bar{z}) \equiv \max_{\mathbf{z}} \left\{ \sum_{i \in \mathcal{M}} v_i(z_i, \theta_i^M) : z_i \in Z_i(\theta_i^M) \text{ for all } i \in \mathcal{M}, \sum_{i \in \mathcal{M}} z_i = \bar{z} \right\}.$$

It is clear that the maximum is attained for all such $\bar{z} \in \sum_i Z_i(\theta_i^M)$, since the constraint set is nonempty and compact, and the objective function is continuous. Convexity of the individual feasible sets and concavity of valuations imply that $\hat{g}$ is concave on $\sum_i Z_i(\theta_i^M)$. Choose $\mathbf{z}^* \in \mathbf{Z}(\theta^M)$ attaining $\hat{g}(\mathbf{0})$.

Now consider the convex set

$$C = \left\{ (\bar{z}, \bar{m}) \in \sum_i Z_i(\theta_i^M) \times \mathbb{R} : \bar{m} \geq \hat{g}(\mathbf{0}) - \hat{g}(\bar{z}) \right\}.$$

The point $(\mathbf{0}, 0)$ belongs to the boundary of C . By the Supporting Hyperplane Theorem, there exists a nonzero vector $(\bar{p}, p_m) \in \mathbb{R}^L \times \mathbb{R}$ such that

$$\bar{p} \cdot \bar{z} + p_m \bar{m} \geq 0, \quad \text{for all } (\bar{z}, \bar{m}) \in C.$$

Since $(\mathbf{0}, 1)$ belongs to C , we have $p_m \geq 0$. If $p_m = 0$, the preceding inequality gives $\bar{p} \cdot \bar{z} \geq 0$ for all $\bar{z} \in \sum_i Z_i(\theta_i^M)$. But from [Assumption 1\(d\)](#), we have that $\mathbf{0} \in \text{int} \sum_i Z_i(\theta_i^M)$, so this requires $\bar{p} = \mathbf{0}$, a contradiction. Thus $p_m > 0$. Normalizing the price of the numeraire to one and writing $p^* = \bar{p}/p_m$, we obtain

$$\hat{g}(\bar{z}) \leq \hat{g}(\mathbf{0}) + p^* \cdot \bar{z}, \quad \text{for all } \bar{z} \in \sum_i Z_i(\theta_i^M).$$

Fix any individual $i \in \mathcal{M}$ and any $z'_i \in Z_i(\theta_i^M)$. Replacing z_i^* by z'_i in $\mathbf{z}^*$ and using the preceding inequality gives

$$v_i(z'_i, \theta_i^M) + \sum_{j \neq i} v_j(z_j^*, \theta_j^M) \leq \hat{g}(z'_i - z_i^*) \leq \hat{g}(\mathbf{0}) + p^* \cdot (z'_i - z_i^*).$$

⁷Indeed, if commodity ℓ could not be supplied, then $z_{i\ell} \geq 0$ for every $i \in \mathcal{M}$ and every $z_i \in Z_i(\theta_i^M)$. The aggregate trading set would then lie in the half-space $\{\bar{z} \in \mathbb{R}^L : \bar{z}_\ell \geq 0\}$, contradicting [Assumption 1\(d\)](#).

⁸See for example [Rockafellar \(1970, Theorem 23.4\)](#).

Since $\hat{g}(\mathbf{0}) = \sum_{j \in \mathcal{M}} v_j(z_j^*, \theta_j^M)$, rearranging the above expression results in

$$v_i(z_i^*, \theta_i^M) - p^* \cdot z_i^* \geq v_i(z_i', \theta_i^M) - p^* \cdot z_i'.$$

For $z_i' \notin Z_i(\theta_i^M)$, the same inequality holds because $v_i(z_i', \theta_i^M) = -\infty$. Thus (z^*, p^*) satisfies [Definition 5](#). Finally, the monetary positions $m_i^* = -p^* \cdot z_i^*$ satisfy $\sum_i m_i^* = 0$, so the market for the numeraire also clears. $\square$

The following is well known (e.g., [Makowski and Ostroy, 1995](#), Proposition 1): local market equilibria produce efficient allocations.

Proposition 5. *If (z^*, p^*) is a market equilibrium at occupational profile $\theta^M \in \Theta^M$, then z^* maximizes $\sum_{i \in \mathcal{M}} v_i(z_i, \theta_i^M)$ on $Z(\theta^M)$, and thus is Pareto efficient at θ^M .*

Equilibrium prices need not be unique, and individual equilibrium payoffs depend on which one prevails. As it turns out, this issue affects existence of occupational equilibria in the market. For now, in order to provide a formal definition of what we mean by occupational equilibrium in a market economy, we follow [Makowski and Ostroy \(1995\)](#) in positing a *price selection mapping*: a commonly known function $p: \Theta^M \rightarrow \mathbb{R}^L$ assigning, to each occupational profile θ^M , an equilibrium price that is correctly anticipated by all individuals when choosing their occupations. Write $z(\theta^M) = (z_i(\theta^M))$ for an associated equilibrium trade profile. Individual i 's *market payoff* at θ^M is then

$$\begin{aligned} \bar{\pi}_i(\theta^M) &\equiv v_i(z_i(\theta^M), \theta_i^M) - p(\theta^M) \cdot z_i(\theta^M) \\ &= \max_{z \in Z_i(\theta_i^M)} \{v_i(z, \theta_i^M) - p(\theta^M) \cdot z\}. \end{aligned} \tag{5}$$

An *occupational equilibrium in the market* is then a profile of occupational choices that gives no individual incentives to deviate unilaterally.

Definition 6. *An occupational equilibrium in the market is a profile $\theta^{M*} = (\theta_i^{M*}) \in \Theta^M$ such that, for each individual $i \in \mathcal{M}$ and each alternative occupation $\theta_i^M \in \Theta_i^M$,*

$$\bar{\pi}_i(\theta_i^{M*}, \theta_{-i}^{M*}) \geq \bar{\pi}_i(\theta_i^M, \theta_{-i}^{M*}). \tag{6}$$

4.2. Existence of Occupational Equilibria in the Market

An occupational equilibrium in the market is a Nash equilibrium of the game in which individuals choose occupations and anticipate the payoffs from a local market equilibrium, taking the price selection mapping as part of the structure of the game (and thus commonly known). Local market equilibria exist at every occupational profile, but this does not in itself establish existence of an occupational equilibrium, even when [Assumption 1](#) is imposed. A key in obtaining existence of occupational equilibria in the market is the possibility of securing a continuous price selection mapping.

For each occupational profile $\theta^M \in \Theta^M$, define the market economy *equilibrium price correspondence* $P: \Theta^M \rightrightarrows \mathbb{R}^L$ by

$$P(\theta^M) \equiv \{p \in \mathbb{R}^L : (z, p) \text{ is a market equilibrium at } \theta^M, \text{ for some } z\}. \quad (7)$$

The next result is the market counterpart of [Theorem 1](#). As before, we assume that [Assumption 1](#), as applied to $\mathcal{M} \neq \emptyset$, continues to hold.

Theorem 2. *Suppose there exists a continuous selection $p: \Theta^M \rightarrow \mathbb{R}^L$ of the equilibrium price correspondence P .*

- (a) *For every $i \in \mathcal{M}$, the payoff function $\bar{\pi}_i: \Theta^M \rightarrow \mathbb{R}$ in [Equation 5](#) is continuous.*
- (b) *If, in addition, for every individual $i \in \mathcal{M}$ and every $\theta_{-i}^M \in \times_{j \neq i} \Theta_j^M$, the set of maximizers of $\theta_i^M \mapsto \bar{\pi}_i(\theta_i^M, \theta_{-i}^M)$ over Θ_i^M is convex, then a pure occupational equilibrium in the market, in the sense of [Definition 6](#), exists.*

Proof. (a) From [Assumption 1\(b\)](#), for each $i \in \mathcal{M}$, the correspondence $Z_i: \Theta_i^M \rightrightarrows \mathbb{R}^L$ is continuous and has nonempty and compact values. [Assumption 1\(c\)](#) and the continuity of the price selection function p imply that i 's objective function in [Equation 5](#), understood as a parametric optimization problem, is finite and continuous on its graph. Berge's Theorem of the Maximum therefore implies that $\bar{\pi}_i$ is continuous on Θ^M .

- (b) Define individual i 's best reply correspondence

$$BR_i(\theta_{-i}^M) \equiv \operatorname{argmax} \{ \bar{\pi}_i(\theta_i^M, \theta_{-i}^M) : \theta_i^M \in \Theta_i^M \},$$

and define also the product correspondence

$$BR(\theta^M) \equiv \times_{i \in \mathcal{M}} BR_i(\theta_{-i}^M).$$

The arguments used in the proof of [Theorem 1](#) apply here too, and thus Kakutani's Fixed Point Theorem provides an occupational profile $\theta^{M*} \in \Theta^M$ such that $\theta^{M*} \in BR(\theta^{M*})$. This is a pure occupational equilibrium in the market economy. $\square$

Notice that our existence result in [Theorem 2\(b\)](#) does not require strict concavity of individual valuations, strict convexity of individual feasible trade sets, or a continuous selection of equilibrium trade profiles. At a given occupation and price, individual payoffs are the values of the corresponding maximization problems; continuity of these values follows directly from Berge's Theorem once a continuous price selection function is in place. Of course, this is an additional requirement. In particular, the selected equilibrium prices may change discontinuously with occupational choices even under [Assumption 1](#).⁹ On the other hand, our setting is sufficiently structured to guarantee some properties of the equilibrium price correspondence P , and points to ways of choosing a continuous selection. We turn our attention to these issues now.

⁹[Makowski and Ostroy \(1995, footnote 7\)](#) point out that concavity of individual valuations alone does not guarantee existence of an occupational equilibrium.

Proposition 6. *The equilibrium price correspondence $P: \Theta^M \rightrightarrows \mathbb{R}^L$ is upper hemicontinuous and has nonempty, compact and convex values that are contained in a common compact set.*

Proof. Define $\mu: \mathbb{R}^L \times \Theta^M \rightarrow \mathbb{R}$ by

$$\mu(p, \theta^M) \equiv \sum_{i \in \mathcal{M}} \max \left\{ v_i(z_i, \theta_i^M) - p \cdot z_i : z_i \in Z_i(\theta_i^M) \right\}.$$

Berge's Theorem of the Maximum implies that μ is continuous. Notice that μ is also convex in p for fixed θ^M , since it is the sum of maxima of affine functions (in prices).

Fix an occupational profile θ^M and a local market equilibrium (z^*, p^*) . For any $p \in \mathbb{R}^L$, individual optimality and feasibility of z^* show that

$$\mu(p, \theta^M) \geq \sum_{i \in \mathcal{M}} v_i(z_i^*, \theta_i^M) = \mu(p^*, \theta^M).$$

Thus, p^* minimizes $\mu(\cdot, \theta^M)$. Conversely, if $p' \in \mathbb{R}^L$ minimizes $\mu(\cdot, \theta^M)$, then it must be that for all $i \in \mathcal{M}$, the trade position z_i^* maximizes $v_i(z_i, \theta_i^M) - p' \cdot z_i$ on $Z_i(\theta_i^M)$. Otherwise, using feasibility of the trade profile z^* , one has

$$\mu(p^*, \theta^M) = \sum_{i \in \mathcal{M}} v_i(z_i^*, \theta_i^M) - p' \cdot \sum_{i \in \mathcal{M}} z_i^* < \mu(p', \theta^M),$$

a contradiction. This shows that if p' minimizes $\mu(\cdot, \theta^M)$, then (z^*, p') is also a local market equilibrium. Consequently, we can write

$$P(\theta^M) = \arg \min \{ \mu(p, \theta^M) : p \in \mathbb{R}^L \}.$$

We conclude that the equilibrium price correspondence P has values that are nonempty, by the existence of local market equilibria, and convex, by convexity of $\mu(\cdot, \theta^M)$.

We next establish a uniform bound on equilibrium prices. Let $A(\theta^M) = \sum_{i \in \mathcal{M}} Z_i(\theta_i^M)$. Seen as a correspondence defined on Θ^M , A is continuous and has compact, convex values. The compactness of Θ^M and [Assumption 1\(d\)](#) imply that there is some $r > 0$ for which

$$\{ \bar{z} \in \mathbb{R}^L : \|\bar{z}\| \leq r \} \subseteq A(\theta^M), \quad \text{for all } \theta^M \in \Theta^M.$$

By [Assumption 1\(a\)](#) and (b), the graph of the individual trade correspondence Z_i is compact, for all $i \in \mathcal{M}$. Since the market economy has finitely many individuals, [Assumption 1\(c\)](#) now furnishes a constant $K > 0$ such that

$$\sum_{i \in \mathcal{M}} |v_i(z_i, \theta_i^M)| \leq K,$$

for every occupational profile $\theta^M = (\theta_i^M)$ and every individually feasible trade profile $z = (z_i)$. For $p = 0$, note we have $\mu(0, \theta^M) \leq K$. For $p \neq 0$, choose individual trade positions that aggregate to $-r p / \|p\|$. With such a trade profile

$$\mu(p, \theta^M) \geq r \|p\| - K.$$

Thus, for any occupational profile θ^M , a price $p \in \mathbb{R}^L$ minimizing $\mu(\cdot, \theta^M)$ satisfies $\|p\| \leq 2K/r$. We may therefore restrict, without loss of generality, the minimization defining the equilibrium price correspondence P to one common compact ball. Berge's Theorem of the Maximum gives the upper hemicontinuity and the compact value property for this correspondence. $\square$

If P is in addition lower hemicontinuous, then it is a continuous correspondence and thus admits a continuous selection (Michael, 1956). Under such a selection, convex occupational best replies as in Theorem 2(b) ensure existence of a pure occupational equilibrium. An alternative route to imposing this extra condition on the endogenous price correspondence is to add conditions on primitives so that equilibrium prices are unique. If there exists a unique equilibrium price vector for each occupational profile, the correspondence P of Equation 7 is singleton-valued. Proposition 6 now shows that we can treat it as a continuous equilibrium price function. Uniqueness can be obtained, for example, from the familiar supporting-price argument underlying the analysis of Makowski and Ostroy (1987). It is enough that, at each occupational profile, some trade profile maximizing the sum of valuations gives an interior trade to an individual whose valuation is differentiable at that trade.

Proposition 7. *Suppose that, for every occupational profile $\theta^M \in \Theta^M$, there exist an individual $j \in \mathcal{M}$ and a trade profile*

$$\hat{z} \in \arg \max \left\{ \sum_{i \in \mathcal{M}} v_i(z_i, \theta_i^M) : z \in Z(\theta^M) \right\}$$

such that $\hat{z}_j \in \text{int } Z_j(\theta_j^M)$ and moreover $v_j(\cdot, \theta_j^M)$ is differentiable at $\hat{z}_j$. Then equilibrium prices are unique at every occupational profile, and all local market equilibria are payoff equivalent.

Proof. Fix θ^M and choose $\hat{z}$ and j as in the statement of the proposition. For any $p \in P(\theta^M)$, let (z, p) be any local market equilibrium. Individual optimal behavior gives

$$v_i(z_i, \theta_i^M) - p \cdot z_i \geq v_i(\hat{z}_i, \theta_i^M) - p \cdot \hat{z}_i, \quad \text{for all } i \in \mathcal{M}.$$

Summing these inequalities across individuals, and using the market clearing condition, the monetary terms on both sides of the aggregate expression cancel. Since $\hat{z}$ maximizes the sum of valuations, the sums on both sides must be equal. Each individual inequality is therefore an equality, so $(\hat{z}, p)$ is also a local market equilibrium that generates the same individual equilibrium payoff to each $i \in \mathcal{M}$.

Since $\hat{z}_j$ is an interior point of $Z_j(\theta_j^M)$, the differentiability of her valuation implies that

$$p = \nabla_z v_j(\hat{z}_j, \theta_j^M) \quad \text{for every } p \in P(\theta^M).$$

Since θ^M was arbitrary, the equilibrium price correspondence is singleton-valued at every occupational profile. $\square$

Only one trade profile maximizing the sum of valuations and one individual need satisfy the condition at each occupational profile. Both may depend on θ^M . The individual's marginal valuation at her interior trade determines the equilibrium price, while continuity follows from upper hemicontinuity of P and uniqueness. Neither the individual nor the maximizing trade profile is required to be selected continuously.

5. The Market and the Jungle Compared

The results obtained so far identify the allocations and payoffs that individuals in the society anticipate when choosing occupations in the market and in the jungle. Suppose that [Assumption 1](#) holds and, in addition, individual valuations are strictly concave in trade positions, individual feasible trade sets are strictly convex, and the interiority and differentiability conditions in [Proposition 7](#) are in place. In this case, the market has a unique local equilibrium at each occupational profile. Likewise, at each occupational profile, the jungle has a unique local equilibrium generated by the serial dictatorship procedure for each hierarchy. Both outcomes determine, uniquely, individual market payoffs and individual expected jungle payoffs. Moreover, these payoff functions are continuous in their respective occupational profiles. When the sets of individual best replies are convex, [Theorem 1](#) and [Theorem 2](#) show existence of pure occupational equilibria in both economies.¹⁰

A main goal in our work is to consider the coexistence of the market and the jungle as a social equilibrium, where all individuals in our society decide which economy to join before making occupational choices. As a first pass, in this section we compare the welfare performance of the market and the jungle as two economic institutions operating on a level playing field. We first hold occupational choices fixed and use aggregate payoffs, i.e., the sum of individual payoffs generated by the market and by the jungle, respectively, as a yardstick. We next consider performance of each economy at its occupational equilibria.

Throughout this section, we organize the entire society once as a market and once as a jungle. Namely, we assume $\mathcal{M} = \mathcal{S} = \mathcal{J}$, and further $\Theta_i^{\mathcal{M}} = \Theta_i = \Theta_i^{\mathcal{J}}$ for every individual $i \in \mathcal{S}$.¹¹ This makes our comparisons here purely institutional: the two economies face the same population, same occupational opportunities, valuations and feasible trade sets. Any difference in performance is therefore due to the institutional rules, and not to occupational opportunities, technology, or preferences. An important institutional difference is that the market supports enforceable transfers of the numeraire, whereas the jungle allocates non-monetary goods through the hierarchy without allowing for monetary compensations.

5.1. Profile by Profile Comparison

Recall that the local equilibria in the jungle selected by the serial dictatorship procedure are Pareto efficient according to [Definition 2](#). Local market equilibria are also Pareto efficient in that sense; in addition, they satisfy the stronger property of maximizing the sum of individual valuations over feasible trade profiles ([Proposition 5](#)). The distinction, of course, comes from the fact that [Definition 2](#) assumes monetary holdings among individuals fixed, whereas the market permits transfers of the numeraire (as long as the money market clears).

Since preferences are quasilinear in the numeraire, the sum of individual valuations provides a common money-metric measure of economic performance. For that purpose, it is convenient to define

$$g(\theta^\varepsilon) \equiv \max_z \left\{ \sum_{i \in \mathcal{E}} v_i(z_i, \theta_i^\varepsilon) : z \in \mathbf{Z}(\theta^\varepsilon) \right\} \quad (8)$$

¹⁰Our existence results do not establish uniqueness of occupational equilibria in either the market or the jungle.

¹¹Except for this section, we treat $\mathcal{M}$ and $\mathcal{J}$ as a partition of $\mathcal{S}$, albeit leaving open the possibility that one of these economies is empty.

as the *maximal gains from trade* at occupational profile $\theta^\mathcal{E}$ in the economy $\mathcal{E} \in \{\mathcal{M}, \mathcal{J}\}$. Because the no-trade option is feasible for every occupational choice, this measure includes the values of no trade at the given occupations; the gains relative to no trade are $g(\theta^\mathcal{E}) - \sum_i v_i(\mathbf{0}, \theta_i^\mathcal{E})$. One readily observes that, under [Assumption 1](#), for every $\theta^\mathcal{E} \in \Theta^\mathcal{E}$ in a nonempty economy, maximal gains from trade in [Equation 8](#) are attained at some trade profile in $\mathbf{Z}(\theta^\mathcal{E})$. If, in addition, $v_i(\cdot, \theta_i^\mathcal{E})$ is strictly concave on $Z_i(\theta_i^\mathcal{E})$ for all $i \in \mathcal{E}$, then maximal gains from trade are attained at a unique trade profile.

We now state our first comparison of the performance of the jungle and the market. For this section, we write $\theta = (\theta_i)$ for a profile in the common set of occupational profiles Θ , and use the superscripts $\mathcal{M}$ and $\mathcal{J}$ only to record which set of rules is applied in each circumstance.

Proposition 8. *Suppose that $\mathcal{M} = \mathcal{S} = \mathcal{J}$, and let $\Theta_i^\mathcal{M} = \Theta_i = \Theta_i^\mathcal{J}$ for every individual $i \in \mathcal{S}$. Then, for every occupational profile $\theta^\mathcal{M} = \theta = \theta^\mathcal{J}$ in Θ , one has*

$$\sum_{i \in \mathcal{S}} \bar{\pi}_i(\theta^\mathcal{J}) \leq \sum_{i \in \mathcal{S}} \bar{\pi}_i(\theta^\mathcal{M}).$$

Proof. Starting with the jungle at occupational profile $\theta^\mathcal{J}$, fix a hierarchy $h \in \mathcal{H}(\mathcal{J})$ and let $\mathbf{z} \in \mathbf{Z}^*(\theta^\mathcal{J}, h)$ be any jungle equilibrium at $(\theta^\mathcal{J}, h)$. By [Corollary 1](#), we have $\pi_{h(n)}(\theta^\mathcal{J}) = v_{h(n)}(z_{h(n)}, \theta_{h(n)}^\mathcal{J})$ for every position $n = 1, \dots, |\mathcal{J}|$ in this hierarchy. Since $\mathbf{z} \in \mathbf{Z}(\theta^\mathcal{J})$,

$$\sum_{n=1}^{|\mathcal{J}|} \pi_{h(n)}(\theta^\mathcal{J}) \leq g(\theta^\mathcal{J}).$$

Averaging over all power relations to obtain individual jungle payoffs, as in [Equation 2](#), preserves the above inequality:

$$\sum_{i \in \mathcal{J}} \bar{\pi}_i(\theta^\mathcal{J}) = \sum_{h \in \mathcal{H}(\mathcal{J})} \frac{1}{|\mathcal{J}|!} \sum_{n=1}^{|\mathcal{J}|} \pi_{h(n)}(\theta^\mathcal{J}) \leq g(\theta^\mathcal{J}). \quad (9)$$

Moving to the market at occupational profile $\theta^\mathcal{M}$, we know from [Proposition 4](#) and [Proposition 5](#), that a local market equilibrium exists, and its associated trade profile $\mathbf{z}(\theta^\mathcal{M}) = (z_i(\theta^\mathcal{M}))$ maximizes $\sum_i v_i(z'_i, \theta_i^\mathcal{M})$ over $\mathbf{Z}(\theta^\mathcal{M})$. Thus we obtain $\sum_i v_i(z_i(\theta^\mathcal{M}), \theta_i^\mathcal{M}) = g(\theta^\mathcal{M})$. The market clearing condition gives $\sum_i z_i(\theta^\mathcal{M}) = \mathbf{0}$. Summing individual payoffs, defined in [Equation 5](#), across individuals in the market immediately obtains

$$\sum_{i \in \mathcal{M}} \bar{\pi}_i(\theta^\mathcal{M}) = \sum_{i \in \mathcal{M}} v_i(z_i(\theta^\mathcal{M}), \theta_i^\mathcal{M}) - p(\theta^\mathcal{M}) \cdot \sum_{i \in \mathcal{M}} z_i(\theta^\mathcal{M}) = g(\theta^\mathcal{M}). \quad (10)$$

Recalling that $\mathcal{M} = \mathcal{S} = \mathcal{J}$ and $\theta^\mathcal{M} = \theta = \theta^\mathcal{J}$, the result follows readily from combining [Equation 9](#) and [Equation 10](#). $\square$

[Equation 10](#) is the accounting identity of [Makowski and Ostroy \(1995\)](#) applied to our setting: market payoffs exhaust the maximal gains from trade at every occupational profile. In equilibrium, monetary payments cancel when individual payoffs are added, because the corresponding positions sum to zero. In the jungle, the hierarchical choice of trade positions can leave a smaller sum of valuations even though the selected outcome is Pareto efficient.

This is what [Proposition 8](#) formalizes: at any given occupational profile, the same group of individuals are collectively weakly better off in the market than in the jungle. It does not imply that every individual prefers the market.

We push this comparison one more step and ask when it is the case that individuals are collectively strictly worse off in the jungle. The relevant tension is between the trade position that an individual would choose if constrained only by her own capacities, and those compatible with maximizing the sum of valuations. To explore this, define individual i 's *satiation set* at occupation $\theta_i \in \Theta_i$ by

$$Z_i^{\text{sat}}(\theta_i) \equiv \arg \max \{v_i(z_i, \theta_i) : z_i \in Z_i(\theta_i)\}. \quad (11)$$

This set contains all the trade positions i would choose, given her occupation θ_i , if she faced no constraints other than her own feasible trade set. Note that under our maintained assumptions, $Z_i^{\text{sat}}(\theta_i)$ is nonempty, compact and convex.

Definition 7. Let $\theta^\varepsilon = (\theta_i^\varepsilon) \in \Theta^\varepsilon$ be an occupational profile for a nonempty economy $\mathcal{E} \in \{\mathcal{M}, \mathcal{J}\}$.

(a) Resources are plentiful at θ^ε if

$$\mathbf{0} \in \sum_{i \in \mathcal{E}} Z_i^{\text{sat}}(\theta_i^\varepsilon).$$

(b) Resources allow the satiation of individual $i \in \mathcal{E}$ at θ^ε if

$$\mathbf{0} \in Z_i^{\text{sat}}(\theta_i^\varepsilon) + \sum_{j \neq i} Z_j(\theta_j^\varepsilon).$$

Both conditions in [Definition 7](#) are stated in terms of individual feasible trade sets and valuations; neither refers to an equilibrium object. Resources are plentiful at θ^ε if there exists a trade profile containing a most-preferred trade position for each individual that can be feasibly provided by the economy. Thus, every individual in the economy can be simultaneously satiated. On the other hand, resources allow the satiation of an individual at θ^ε if there exists a feasible trade profile that gives said individual one of her most-preferred trades, without requiring that the others be satiated. When the latter condition holds for every individual, we say that resources allow the satiation of all individuals separately.

If resources are plentiful, the role played by either economic institution, the anonymous market or the non-anonymous jungle, is essentially the same: coordination of economic activities, without arbitration of wants by competitive prices or hierarchical choices. Indeed, the serial dictatorship procedure can satiate every individual under every hierarchy. Thus, if resources are plentiful at $\theta^\mathcal{M} = \theta = \theta^\mathcal{J}$, and $\mathcal{M} = \mathcal{S} = \mathcal{J}$, we have

$$\sum_{i \in \mathcal{S}} \bar{\pi}_i(\theta^\mathcal{M}) = g(\theta) = \sum_{i \in \mathcal{S}} \bar{\pi}_i(\theta^\mathcal{J}).$$

In contrast, individual satiation need not be compatible with maximizing the sum of valuations. If an individual can be satiated in the jungle economy, but no trade profile maximizing aggregate surplus satiates her, placing her first in the hierarchy produces a strictly smaller aggregate surplus. Since every hierarchy has positive probability, one such individual is enough for the performance difference between the market and the jungle to be strict.

Proposition 9. For an occupational profile $\theta^\mathcal{J} \in \Theta^\mathcal{J}$ in a nonempty jungle, the following hold.

- (a) If $g(\theta^\mathcal{J}) > \sum_{i \in \mathcal{J}} \bar{\pi}_i(\theta^\mathcal{J})$, then resources in the jungle are not plentiful at $\theta^\mathcal{J}$.
- (b) If resources allow the satiation of some $i \in \mathcal{J}$ at $\theta^\mathcal{J}$, but no feasible trade profile attaining $g(\theta^\mathcal{J})$ contains a trade position for i that belongs to $Z_i^{\text{sat}}(\theta_i^\mathcal{J})$, then $g(\theta^\mathcal{J}) > \sum_{j \in \mathcal{J}} \bar{\pi}_j(\theta^\mathcal{J})$.
- (c) In particular, the strict inequality holds if $g(\theta^\mathcal{J})$ is attained at a unique trade profile, resources are not plentiful, and resources allow the satiation of all individuals separately.

Proof. (a) We show the contrapositive. When resources are plentiful at $\theta^\mathcal{J}$, there exists a feasible trade profile $\mathbf{z}^{\text{sat}} = (z_i^{\text{sat}}) \in \mathbf{Z}(\theta^\mathcal{J})$ such that $z_i^{\text{sat}} \in Z_i^{\text{sat}}(\theta_i^\mathcal{J})$ for every $i \in \mathcal{J}$. This profile maximizes each individual's valuation over her own feasible trade set, and thus

$$g(\theta^\mathcal{J}) = \sum_{i \in \mathcal{J}} v_i(z_i^{\text{sat}}, \theta_i^\mathcal{J}).$$

Fix any hierarchy $h \in \mathcal{H}(\mathcal{J})$. Since $\mathbf{z}^{\text{sat}}$ belongs to $\mathbf{Z}(\theta^\mathcal{J}) = \mathbf{Z}_{h(0)}(\theta^\mathcal{J})$ and maximizes the first dictator's valuation, it belongs to $\mathbf{Z}_{h(1)}(\theta^\mathcal{J})$. Recursively, if $\mathbf{z}^{\text{sat}}$ belongs to $\mathbf{Z}_{h(n-1)}(\theta^\mathcal{J})$, it also maximizes individual $h(n)$'s valuation over this constraint set, and hence belongs to $\mathbf{Z}_{h(n)}(\theta^\mathcal{J})$. This argument shows that $\mathbf{z}^{\text{sat}} \in \mathbf{Z}^*(\theta^\mathcal{J}, h)$. From [Corollary 1](#), every local jungle equilibrium in $\mathbf{Z}^*(\theta^\mathcal{J}, h)$ gives each individual her satiation payoff. This holds under every hierarchy, so averaging gives

$$\sum_{i \in \mathcal{J}} \bar{\pi}_i(\theta^\mathcal{J}) = \sum_{i \in \mathcal{J}} v_i(z_i^{\text{sat}}, \theta_i^\mathcal{J}) = g(\theta^\mathcal{J}).$$

(b) Suppose resources allow the satiation of $i \in \mathcal{J}$. Fix a hierarchy $h \in \mathcal{H}(\mathcal{J})$ with $h(1) = i$, and let $\hat{\mathbf{z}} \in \mathbf{Z}^*(\theta^\mathcal{J}, h)$. Since i is the most powerful individual, her maximal valuation over $Z_i(\theta_i^\mathcal{J})$ can be attained by a feasible trade profile. Therefore, the first optimization problem in the recursion of [Proposition 1](#) with the hierarchy h gives $\hat{z}_i \in Z_i^{\text{sat}}(\theta_i^\mathcal{J})$. Since no feasible trade profile attaining $g(\theta^\mathcal{J})$ includes $\hat{z}_i$, we have

$$\sum_{j \in \mathcal{J}} v_j(\hat{z}_j, \theta_j^\mathcal{J}) < g(\theta^\mathcal{J}).$$

This holds under every hierarchy that places individual i in the first position. Under every other hierarchy, the aggregate payoff is at most $g(\theta^\mathcal{J})$. Consequently,

$$\sum_{j \in \mathcal{J}} \bar{\pi}_j(\theta^\mathcal{J}) = \frac{1}{|\mathcal{J}|!} \sum_h \sum_{n=1}^{|\mathcal{J}|} \pi_{h(n)}(\theta^\mathcal{J}) < g(\theta^\mathcal{J}).$$

(c) Let $\bar{\mathbf{z}} = (\bar{z}_j)$ be the unique trade profile attaining $g(\theta^\mathcal{J})$. Since resources are not plentiful at $\theta^\mathcal{J}$, for some individual $i \in \mathcal{J}$ it must be true that $\bar{z}_i \notin Z_i^{\text{sat}}(\theta_i^\mathcal{J})$. Otherwise, $\bar{\mathbf{z}}$ would constitute a feasible trade profile satiating all individuals, which is not possible. On the other hand, resources allow the satiation of i separately. Since $\bar{\mathbf{z}}$ uniquely attains $g(\theta^\mathcal{J})$, we conclude that no trade profile attaining $g(\theta^\mathcal{J})$ satiates her. The argument employed to prove (b) now applies. $\square$

Proposition 9(b) isolates what hierarchical, non-anonymous institutions cost in terms of welfare performance. Under individual satiation, there is an individual in the jungle who, when operating in the top position in the hierarchy, can coerce others to achieve her satiation point. But her objective is in direct conflict with the maximization of aggregate welfare, understood as the sum of valuations. The jungle has no mechanism to offer her compensation for choosing a trade profile that maximizes gains from trade instead of her satiation trade position. The serial dictatorship outcome remains Pareto efficient. Under the conditions of part (b), placing this individual first selects an allocation that does not maximize aggregate welfare.

The following sufficient condition makes the conflict identified in **Proposition 9** explicit in terms of primitives: only one individual is required to have a unique interior satiation trade and a valuation that is differentiable there. The valuations of the other individuals need not be strictly concave or differentiable.

Corollary 3. *Suppose resources in the jungle are not plentiful at occupational profile $\theta^J = (\theta_i^J) \in \Theta^J$. Suppose in addition that resources allow the satiation of an individual $i \in J$ whose valuation $v_i(\cdot, \theta_i^J)$ admits a unique interior satiation trade, i.e., $Z_i^{\text{sat}}(\theta_i^J) = \{z_i^{\text{sat}}\}$, with $z_i^{\text{sat}} \in \text{int } Z_i(\theta_i^J)$. If $v_i(\cdot, \theta_i^J)$ is differentiable at z_i^{sat} , then*

$$g(\theta^J) > \sum_{j \in J} \bar{\pi}_j(\theta^J).$$

Proof. Let $\hat{z} = (\hat{z}_j) \in Z(\theta^J)$ be any feasible trade profile satiating individual i , so $\hat{z}_i = z_i^{\text{sat}}$. Since resources are not plentiful, there exists some individual $j \neq i$ with $\hat{z}_j \notin Z_j^{\text{sat}}(\theta_j^J)$. Choose any $z_j^{\text{sat}} \in Z_j^{\text{sat}}(\theta_j^J)$ and let $\delta = v_j(z_j^{\text{sat}}, \theta_j^J) - v_j(\hat{z}_j, \theta_j^J) > 0$.

For $\epsilon > 0$, define a trade profile $z(\epsilon)$ by

$$z_i(\epsilon) = z_i^{\text{sat}} - \epsilon(z_j^{\text{sat}} - \hat{z}_j), \quad z_j(\epsilon) = \hat{z}_j + \epsilon(z_j^{\text{sat}} - \hat{z}_j),$$

and $z_k(\epsilon) = \hat{z}_k$ for all $k \neq i, j$. For sufficiently small $\epsilon > 0$, this trade profile is feasible. Individual i 's trade remains in her feasible set because z_i^{sat} is an interior point, individual j 's trade is feasible by the convexity of $Z_j(\theta_j^J)$, and the changes in their trades sum to zero.

Since $z_i^{\text{sat}} = \hat{z}_i$ is an interior maximizer and $v_i(\cdot, \theta_i^J)$ is differentiable at this trade position, we have $\nabla_z v_i(z_i^{\text{sat}}, \theta_i^J) = \mathbf{0}$, and thus we can write

$$v_i(z_i(\epsilon), \theta_i^J) - v_i(\hat{z}_i, \theta_i^J) = o(\epsilon).$$

Concavity of $v_j(\cdot, \theta_j^J)$ implies that

$$v_j(z_j(\epsilon), \theta_j^J) - v_j(\hat{z}_j, \theta_j^J) \geq \epsilon \delta.$$

Hence, for sufficiently small $\epsilon > 0$,

$$\sum_{k \in J} v_k(z_k(\epsilon), \theta_k^J) - \sum_{k \in J} v_k(\hat{z}_k, \theta_k^J) \geq \epsilon \delta + o(\epsilon) > 0.$$

Hence no feasible trade profile satiating i attains $g(\theta^J)$. **Proposition 9(b)** now applies. $\square$

5.2. Equilibrium Comparison

[Proposition 8](#), and the strict version that follows either [Proposition 9](#) or [Corollary 3](#), compare the welfare performance of the market and the jungle economies profile by profile, not at their respective equilibria. But individuals optimally choose their occupations in each economy, anticipating the optimizing behavior of others in each context. And the rules that regulate economic interactions for the market and the jungle are different. In the market, each individual best responds to the market payoffs given in [Equation 5](#), which value an occupation at what it earns net of prices. In the jungle, each individual best responds to the expected serial dictatorship payoff in [Equation 2](#), which values an occupation at what its holder can take given her position in the hierarchy, averaged over all the positions she may draw. So, in general, the occupational equilibria of the two economies differ, even in the common environment we analyze in this section, where the economies are populated by the same individuals, and individuals have the same occupational opportunities in either of them.

Our main result in this section states that when the market economy is perfectly competitive, in a sense that we make precise below, it outperforms the jungle economy when both institutions operate at equilibrium. As readers familiar with their work will surely recognize, to obtain our institutional comparison at equilibrium positions we adapt ideas about the meaning of perfect competition in [Makowski and Ostroy \(1995\)](#) and [Makowski and Ostroy \(1987\)](#). In the next definition, we take $p: \Theta^M \rightarrow \mathbb{R}^L$ to be a continuous selection of the equilibrium price correspondence P defined in [Equation 7](#). Local market equilibria are those associated with this selection.

Definition 8. An occupational equilibrium in the market $\theta^{M*} = (\theta_i^{M*}) \in \Theta^M$ is perfectly competitive if there exists a price vector $p^* \in \mathbb{R}^L$ such that

$$p^* = p(\theta_i^M, \theta_{-i}^{M*}), \quad \text{for all } \theta_i^M \in \Theta_i^M, \text{ all } i \in \mathcal{M}.$$

If an occupational equilibrium in the market is perfectly competitive, no individual can influence the equilibrium prices selected by p through an uncoordinated change in her occupational choices. Thus, under perfect competition, every individual acts as a *price-taker*, given others' equilibrium occupational choices, and cannot affect prices either by changing what she trades or by changing what she selects as occupation.

Theorem 3. Suppose that $\mathcal{M} = \mathcal{S} = \mathcal{J}$ and $\Theta_i^M = \Theta_i = \Theta_i^J$ for every individual $i \in \mathcal{S}$. Let $\theta^{M*} = (\theta_i^{M*})$ be any occupational equilibrium for the market and $\theta^{J*} = (\theta_i^{J*})$ be any occupational equilibrium for the jungle. If θ^{M*} is perfectly competitive, then

$$\sum_{i \in \mathcal{S}} \bar{\pi}_i(\theta^{J*}) \leq \sum_{i \in \mathcal{S}} \bar{\pi}_i(\theta^{M*}).$$

Proof. Starting with the market economy, let p^* be the price vector associated with the perfectly competitive occupational equilibrium θ^{M*} . Now take any occupational profile $\theta^M \in \Theta^M$ and choose $\bar{z} = (\bar{z}_i) \in \mathbf{Z}(\theta^M)$ attaining $g(\theta^M)$. For every $i \in \mathcal{M}$, since $\bar{z}_i \in Z_i(\theta_i^M)$, individual optimal behavior in the market at $(\theta_i^M, \theta_{-i}^{M*})$ with equilibrium prices p^* implies

$$\bar{\pi}_i(\theta_i^M, \theta_{-i}^{M*}) \geq v_i(\bar{z}_i, \theta_i^M) - p^* \cdot \bar{z}_i.$$

Adding the above inequality across all individuals in $\mathcal{M}$, using $\sum_i \bar{z}_i = \mathbf{0}$, and subtracting $\sum_i \bar{\pi}_i(\boldsymbol{\theta}^{M*}) = g(\boldsymbol{\theta}^{M*})$ from both sides of the resulting expression results in

$$0 \geq \sum_{i \in \mathcal{M}} \left[\bar{\pi}_i(\boldsymbol{\theta}_i^M, \boldsymbol{\theta}_{-i}^{M*}) - \bar{\pi}_i(\boldsymbol{\theta}^{M*}) \right] \geq g(\boldsymbol{\theta}^M) - g(\boldsymbol{\theta}^{M*}),$$

where the first inequality follows from the fact that $\boldsymbol{\theta}^{M*}$ is an occupational equilibrium in the market. Since $\boldsymbol{\theta}^M$ was an arbitrary occupational profile, we conclude that $\boldsymbol{\theta}^{M*}$ maximizes gains from trade over all occupational profiles.

Going to the jungle, every local jungle equilibrium allocation at $\boldsymbol{\theta}^{J*}$ is feasible and thus has aggregate surplus bounded above by $g(\boldsymbol{\theta}^{J*})$. Since the two institutions face the same occupational opportunities and the same population, i.e., $\boldsymbol{\Theta}^M = \boldsymbol{\Theta} = \boldsymbol{\Theta}^J$, averaging over hierarchies and applying the preceding conclusion yields

$$\sum_{i \in \mathcal{S}} \bar{\pi}_i(\boldsymbol{\theta}^{J*}) \leq g(\boldsymbol{\theta}^{J*}) \leq g(\boldsymbol{\theta}^{M*}) = \sum_{i \in \mathcal{S}} \bar{\pi}_i(\boldsymbol{\theta}^{M*}). \quad \square$$

Of course, under the additional conditions of [Proposition 9](#) or [Corollary 3](#), the conclusion of [Theorem 3](#) can be strengthened to show that aggregate surplus in a perfectly competitive occupational equilibrium in the market is strictly higher than aggregate surplus in the corresponding equilibrium in the jungle.

To gain intuition for our comparison results, let

$$\Delta(\boldsymbol{\theta}^J) \equiv g(\boldsymbol{\theta}^J) - \sum_{i \in \mathcal{J}} \bar{\pi}_i(\boldsymbol{\theta}^J) \geq 0 \quad \text{and} \quad g^* \equiv \max_{\boldsymbol{\theta} \in \boldsymbol{\Theta}} g(\boldsymbol{\theta}).$$

The first expression can be understood as the jungle's *trade wedge* at $\boldsymbol{\theta}^J$. It is the difference between maximal gains from trade at $\boldsymbol{\theta}^J$ and the expected aggregate payoffs generated by the serial dictatorship. It is always non-negative by [Proposition 8](#), and strictly positive under the circumstances described in [Proposition 9](#) and [Corollary 3](#). The term g^* captures the largest maximal gains from trade that can be generated by an occupational profile in $\boldsymbol{\Theta}$.

Let $\boldsymbol{\Theta}^M = \boldsymbol{\Theta} = \boldsymbol{\Theta}^J$ again. Adding and subtracting g^* to the difference $\sum_i \bar{\pi}_i(\boldsymbol{\theta}^{M*}) - \sum_i \bar{\pi}_i(\boldsymbol{\theta}^{J*})$, using [Equation 10](#), and rearranging gives

$$\sum_{i \in \mathcal{S}} \bar{\pi}_i(\boldsymbol{\theta}^{M*}) - \sum_{i \in \mathcal{S}} \bar{\pi}_i(\boldsymbol{\theta}^{J*}) = [g^* - g(\boldsymbol{\theta}^{J*})] - [g^* - g(\boldsymbol{\theta}^{M*})] + \Delta(\boldsymbol{\theta}^{J*}). \quad (12)$$

This identity separates the difference in institutional performance into three components. The first two components respond to equilibrium occupational choices in the jungle and in the market, respectively. Indeed, the *occupational wedge* $g^* - g(\boldsymbol{\theta}^{J*})$ measures the gap between the largest gains from trade that can be attained in this society, and those realized at the occupational equilibrium for the economy $\mathcal{E} \in \{\mathcal{M}, \mathcal{J}\}$. The occupational wedge is by definition non-negative, for any occupational equilibrium in either economy. It captures the failures of each economy to implement, as equilibrium, the occupations that would generate g^* . The third component is the local trade wedge in the jungle at its occupational equilibrium. This is what [Equation 9](#) measures: at $\boldsymbol{\theta}^{J*}$, each selected trade profile is Pareto efficient, but need not maximize the sum of valuations.

The decomposition in Equation 12 settles what our profile by profile comparison in Section 5.1 does, and does not, imply. Even when at any given occupational profile the market performs better than the jungle, as Proposition 8 establishes, it is still possible that at their respective equilibrium occupations, the jungle is collectively better off than the market. Indeed, this would happen if the market occupational wedge $g^* - g(\theta^{M*})$ is sufficiently large. The pointwise comparison of Proposition 8 is compatible with either outcome at equilibrium: the jungle dominates precisely when the market's occupational wedge exceeds the jungle's own occupational wedge together with its local trade wedge. The decomposition in Equation 12 also pins down the effects of perfect competition in the market. If the occupational equilibrium θ^{M*} is perfectly competitive, then the market occupational wedge vanishes, and the difference between market performance and jungle performance at equilibrium unambiguously favors the market. This is the message of Theorem 3.

Both comparisons concern aggregate payoffs: at a common occupational profile, and at occupational equilibria when the market equilibrium is perfectly competitive, aggregate market payoffs are at least as high as aggregate jungle payoffs. It is still possible, of course, for some individuals to be better off (even strictly) in the jungle economy. These individual comparisons are relevant for a future extension of our work, where we allow the market and the jungle to coexist with endogenous sizes and composition.

Nonetheless, our level-playing-field setting in this section permits individual comparison in a special case that is of independent interest. From Assumption 1(b), the null trade position $\mathbf{0}$ is individually feasible for any occupational choice $\theta_i \in \Theta_i$. Therefore, when occupational choices in the market and in the jungle are the same as in the whole society, at any market occupational equilibrium θ^{M*} we have

$$\bar{\pi}_i(\theta^{M*}) \geq \bar{\pi}_i(\theta_i, \theta_{-i}^{M*}) \geq v_i(\mathbf{0}, \theta_i) \quad (13)$$

for any possible deviation $\theta_i \in \Theta_i$, since the null trade position is affordable for i at whatever equilibrium prices prevail in the market following her deviation to θ_i .¹²

Recall also that individual payoffs in the jungle are obtained by taking expectations over the family of all possible hierarchies in $\mathcal{J}$. Given θ^{J*} , let $z_i(\theta^{J*}, h)$ denote the trade position for individual i selected by the hierarchy $h \in \mathcal{H}(\mathcal{J})$ as part of its local jungle equilibrium allocation, and let $z_i^e(\theta^{J*})$ denote its expectation, i.e., $z_i^e(\theta^{J*}) \equiv \mathbb{E}_{\mathcal{H}(\mathcal{J})}[z_i(\theta^{J*}, h)]$.

Proposition 10. *Suppose that $\mathcal{M} = \mathcal{S} = \mathcal{J}$ and $\Theta_i^M = \Theta_i = \Theta_i^J$ for every individual $i \in \mathcal{S}$. Let $\theta^{M*} = (\theta_i^{M*})$ and $\theta^{J*} = (\theta_i^{J*})$ be occupational equilibria in the market and the jungle, respectively.*

- (a) *If individual i 's expected equilibrium allocation in the jungle is $z_i^e(\theta^{J*}) = \mathbf{0}$, then she is weakly better off in the market than in the jungle: $\bar{\pi}_i(\theta^{J*}) \leq \bar{\pi}_i(\theta^{M*})$.*
- (b) *If all individuals in the society are identical, in terms of occupational choices, preferences, and individual feasible trade sets, and if the occupational equilibrium θ^{J*} is symmetric, then every individual is weakly better off in the market than in the jungle.*

Proof. (a) Suppose that $z_i^e(\theta^{J*}) = \mathbf{0}$. Since $v_i(\cdot, \theta_i^{J*})$ is a concave function, it follows from the

¹²Thus, here we are not assuming that the occupational equilibrium θ^{M*} is perfectly competitive.

definition of the expected jungle payoff function in Equation 2 that

$$\begin{aligned}\bar{\pi}_i(\boldsymbol{\theta}^{\mathcal{J}^*}) &= \mathbb{E}_{\mathcal{H}(\mathcal{J})} [v_i(z_i(\boldsymbol{\theta}^{\mathcal{J}^*}, h), \theta_i^{\mathcal{J}^*})] \\ &\leq v_i(z_i^e(\boldsymbol{\theta}^{\mathcal{J}^*}), \theta_i^{\mathcal{J}^*}) \\ &= v_i(\mathbf{0}, \theta_i^{\mathcal{J}^*}) \leq \max \{v_i(\mathbf{0}, \theta_i) : \theta_i \in \Theta_i\} \leq \bar{\pi}_i(\boldsymbol{\theta}^{\mathcal{M}^*}),\end{aligned}$$

where the last inequality follows from the observation we made in Equation 13.

(b) Suppose that $\boldsymbol{\theta}^{\mathcal{J}^*}$ is a symmetric occupational equilibrium in the jungle. Each individual then selects the same occupation, has the same feasible trade set, and has the same valuation function. Relabeling individuals leaves the serial dictatorship problems unchanged. Consequently, the payoff associated with each position in the hierarchy does not depend on the identity of the individual occupying it. Uniform randomization over hierarchies therefore gives every individual the same expected payoff. Indeed, for any hierarchy $h \in \mathcal{H}(\mathcal{J})$, concavity of the common valuation function and market clearing imply, for every individual $i \in \mathcal{J}$,

$$\frac{1}{|\mathcal{J}|} \sum_{j \in \mathcal{J}} v_j(z_j(\boldsymbol{\theta}^{\mathcal{J}^*}, h), \theta_j^{\mathcal{J}^*}) \leq v_i\left(\frac{1}{|\mathcal{J}|} \sum_{j \in \mathcal{J}} z_j(\boldsymbol{\theta}^{\mathcal{J}^*}, h), \theta_i^{\mathcal{J}^*}\right) = v_i(\mathbf{0}, \theta_i^{\mathcal{J}^*}).$$

Taking expectations, the expression on the left equals the average expected jungle payoff and hence, by symmetry, each individual's expected jungle payoff. Thus $\bar{\pi}_i(\boldsymbol{\theta}^{\mathcal{J}^*}) \leq v_i(\mathbf{0}, \theta_i^{\mathcal{J}^*})$. Equation 13 now gives $\bar{\pi}_i(\boldsymbol{\theta}^{\mathcal{J}^*}) \leq \bar{\pi}_i(\boldsymbol{\theta}^{\mathcal{M}^*})$ for each i in the society. $\square$

A few comments regarding Proposition 10 are worth mentioning. First, suppose valuations are strictly concave. Then the comparison in part (a) is strict whenever individual i 's equilibrium trades in the jungle vary across hierarchies. Under the symmetry conditions in part (b), the comparison is strict for every individual whenever the equilibrium jungle trade lottery is non-degenerate. Strict concavity therefore gives a strict welfare comparison when the hierarchy generates uncertainty over individual trades. Under these conditions, when the society is populated by identical individuals, each of them is worse off (and thus also collectively worse off) in a symmetric occupational equilibrium in the jungle than in any occupational equilibrium in the market. Second, this does not assume that the market occupational equilibrium is perfectly competitive. A perfectly competitive occupational equilibrium in the market realizes the largest gains from trade. Instead, what goes on in Proposition 10 is that the symmetric occupational equilibrium in the jungle underperforms any occupational equilibrium in the market.

Third, note that with identical individuals and concave valuations, a symmetric occupational equilibrium in the market can be implemented with zero trade at the selected prices. In this case, the market as an institution performs better than the jungle not because it generates gains from trade, but because it respects the rights of the individuals to choose their null trade positions. Finally, in our model, the fact that identical individuals populate the society does not mean that the only possible occupational equilibria are symmetric. Different occupational choices may generate different marginal valuations of the commodities, and thus both the jungle economy and the market economy can sustain asymmetric equilibria (with ex post differentiation). In particular, the market can support beneficial exchange between individuals who were identical before making occupational decisions.

Some of these points are illustrated in the next example.

Example 3. There are two individuals and two non-monetary commodities. Both individuals, $i = 1, 2$, have the same set of occupations $\Theta_i = [-1, 1]$, the same feasible trade correspondence $Z_i: \Theta_i \rightrightarrows \mathbb{R}^2$ given by

$$Z_i(\theta_i) = \{z_i \in \mathbb{R}^2 : z_{i1}^2 + z_{i2}^2 \leq 9\},$$

and the same valuation function $v_i: \mathbb{R}^2 \times \Theta_i \rightarrow \mathbb{R} \cup \{-\infty\}$, which assigns the value

$$v_i(z_i, \theta_i) = 3 + (1 + \theta_i)z_{i1} + (1 - \theta_i)z_{i2} - \frac{1}{2}(z_{i1}^2 + z_{i2}^2 + \theta_i^2)$$

to any trade that belongs to $Z_i(\theta_i)$ and $-\infty$ otherwise.

Notice that a positive occupation raises the marginal valuation of the first good and lowers that of the second; a negative occupation reverses these effects. The term $\theta_i^2/2$ can be interpreted as the cost of the occupational choice. These two individuals are identical. Moreover, their feasible trade sets remain identical at every occupational profile, although, and this detail is important, different occupational choices induce different marginal values over trades. All the conditions in [Assumption 1](#) are met; in addition, valuations are strictly concave and differentiable in trade and the feasible trade sets are strictly convex.

Starting with the market economy, at an occupational profile $\theta = (\theta_1, \theta_2)$, the unique market equilibrium consists of prices

$$p(\theta) = (1 + \frac{1}{2}(\theta_1 + \theta_2), 1 - \frac{1}{2}(\theta_1 + \theta_2))$$

and trades

$$z_i(\theta) = (\frac{1}{2}(\theta_i - \theta_j), -\frac{1}{2}(\theta_i - \theta_j)), \quad \text{where } j \neq i.$$

These trades are in the interior of the feasible sets, maximize each individual's valuation net of prices, and clear the two commodity markets. Individual i 's market payoff is then

$$\bar{\pi}_i(\theta^M) = 3 + \frac{1}{4}(\theta_i - \theta_j)^2 - \frac{1}{2}\theta_i^2.$$

Individual i 's market payoff function is strictly concave in θ_i . It is readily seen that, for each $\theta_j \in \Theta_j$, its unique maximizer is $\theta_i = -\theta_j$. Hence the market occupational equilibria are all profiles $\theta^M = (\theta, -\theta)$, with $\theta \in [-1, 1]$. At any such profile, each individual obtains a market payoff equal to $3 + \theta^2/2$. In particular, $\theta^{M*} = (1, -1)$ is a market occupational equilibrium with

$$z_1(\theta^{M*}) = (1, -1), \quad z_2(\theta^{M*}) = (-1, 1), \quad \text{and} \quad p(\theta^{M*}) = (1, 1).$$

Each individual supplies one unit of one good and receives one unit of the other, and both obtain a market payoff equal to $7/2$. In contrast, their best market payoff without trade is 3, which is attained at the occupational profile $\theta^{M^0} = (0, 0)$. In this case, the individuals are ex post identical and the market equilibrium involves no trade. Hence, the occupational equilibrium $\theta^{M*} = (1, -1)$ generates a strict gain over the best zero-trade outcome for each individual, because it allows them to realize gains from trade after specialization.

Going to the jungle now, at any occupational profile $\theta = (\theta_1, \theta_2)$, if individual i is first in the

hierarchy, she selects her satiation trade

$$z_i^{\text{sat}}(\theta) = (1 + \theta_i, 1 - \theta_i).$$

The other individual is coerced into taking $-z_i^{\text{sat}}(\theta)$ as her trade. These trades are individually feasible and clear the jungle. Averaging over the two possible hierarchies gives individual i 's jungle expected payoff:

$$\bar{\pi}_i(\theta^J) = 2 - \theta_i \theta_j - \frac{1}{2} \theta_j^2.$$

For fixed θ_j , individual i 's jungle payoff is linear in θ_i . Her best reply correspondence takes the single value -1 when $\theta_j > 0$, 1 when $\theta_j < 0$, and the entire interval $[-1, 1]$ when $\theta_j = 0$. Readily, all best-reply sets are convex, as they are in the market.

The profile $\theta^{J*} = (0, 0)$ is a symmetric jungle occupational equilibrium. At this equilibrium, individual i obtains trade $(1, 1)$ and payoff 4 when she is the first in the hierarchy, and trade $(-1, -1)$ and payoff 0 when she is the second. Consequently,

$$z_i^e(\theta^{J*}) = 0 \quad \text{and} \quad \bar{\pi}_i(\theta^{J*}) = 2, \quad \text{for } i = 1, 2.$$

Comparing θ^{J*} with θ^{M*} and $\theta^{M\circ}$ shows that

$$\bar{\pi}_i(\theta^{J*}) = 2 < 3 = \bar{\pi}_i(\theta^{M\circ}) < \frac{7}{2} = \bar{\pi}_i(\theta^{M*}), \quad i = 1, 2.$$

The first inequality captures the loss from random coercive trade in the jungle equilibrium θ^{J*} relative to the zero-trade market equilibrium $\theta^{M\circ}$. Note that this is purely driven by institutional rules, as in these two equilibria individuals choose the same occupations and are thus ex post identical. The second inequality captures the gain supported by occupational differentiation and voluntary exchange in the market equilibrium θ^{M*} .

There are two other occupational equilibria in this jungle: $(1, -1)$ and $(-1, 1)$, each giving expected payoff $5/2$ to both individuals. Thus, in this example, every market occupational equilibrium is strictly preferred to every jungle occupational equilibrium by each individual: welfare comparison does not depend on selecting the symmetric jungle equilibrium. $\diamond$

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Appendix

A. Payoff Dominance of Local Jungle Equilibria

In this appendix we show that, under an additional assumption, the local jungle equilibria identified in [Proposition 1](#) are weakly payoff dominant. Fix a power relation $h \in \mathcal{H}(\mathcal{J})$ for a nonempty jungle. Recalling that $\mathbf{Z}_{h(0)}(\boldsymbol{\theta}^{\mathcal{J}}) \equiv \mathbf{Z}(\boldsymbol{\theta}^{\mathcal{J}})$, define

$$\mathbf{Y}_{h(n)}(\boldsymbol{\theta}^{\mathcal{J}}) \equiv \mathbf{Z}_{h(n-1)}(\boldsymbol{\theta}^{\mathcal{J}}) \setminus \mathbf{Z}_{h(n)}(\boldsymbol{\theta}^{\mathcal{J}}), \quad \text{for all } n = 1, \dots, |\mathcal{J}|.$$

These are the trade profiles discarded at step n of the recursion in the proof of [Proposition 1](#): they give every stronger individual her payoff in the serial dictatorship, but provide individual $h(n)$ strictly less than $\pi_{h(n)}(\boldsymbol{\theta}^{\mathcal{J}})$.

Assumption 2. An occupational profile $\boldsymbol{\theta}^{\mathcal{J}} \in \boldsymbol{\Theta}^{\mathcal{J}}$ satisfies improvement along the hierarchy $h \in \mathcal{H}(\mathcal{J})$ if, for every $m \in \{1, \dots, |\mathcal{J}| - 1\}$ and every $\hat{\mathbf{z}} \in \mathbf{Y}_{h(m)}(\boldsymbol{\theta}^{\mathcal{J}})$, there exist $n \in \{m + 1, \dots, |\mathcal{J}|\}$ and a feasible trade profile $\tilde{\mathbf{z}} \in \mathbf{Z}(\boldsymbol{\theta}^{\mathcal{J}})$ such that $v_i(\tilde{z}_i, \theta_i^{\mathcal{J}}) = v_i(\hat{z}_i, \theta_i^{\mathcal{J}})$ for all $i \in \mathcal{J}$, $i \neq h(m), h(n)$, and $v_{h(m)}(\tilde{z}_{h(m)}, \theta_{h(m)}^{\mathcal{J}}) > v_{h(m)}(\hat{z}_{h(m)}, \theta_{h(m)}^{\mathcal{J}})$.

The above condition requires an improving deviation only at profiles discarded before the final step of the recursion, and only against some weaker individual. The trade positions of third parties may change, provided their payoffs remain unchanged, as permitted by [Definition 3](#). Under improvements along the hierarchy h , the local equilibria in $\mathbf{Z}^*(\boldsymbol{\theta}^{\mathcal{J}}, h)$ payoff dominate any other jungle equilibrium at $\boldsymbol{\theta}^{\mathcal{J}}$.

Proposition 11. Suppose [Assumption 1](#) holds and fix a power relation $h \in \mathcal{H}(\mathcal{J})$. Suppose the occupational profile $\boldsymbol{\theta}^{\mathcal{J}} \in \boldsymbol{\Theta}^{\mathcal{J}}$ satisfies [Assumption 2](#) relative to h .

(a) If $\hat{\mathbf{z}}$ is a jungle equilibrium for $(\boldsymbol{\theta}^{\mathcal{J}}, h)$, then for all $n = 1, \dots, |\mathcal{J}|$,

$$v_{h(n)}(\hat{z}_{h(n)}, \theta_{h(n)}^{\mathcal{J}}) \leq \pi_{h(n)}(\boldsymbol{\theta}^{\mathcal{J}}). \quad (14)$$

(b) [Equation 14](#) holds with equality for every position $n < |\mathcal{J}|$ in the hierarchy. For the weakest individual, the inequality is strict if and only if $\hat{\mathbf{z}} \notin \mathbf{Z}^*(\boldsymbol{\theta}^{\mathcal{J}}, h)$.

(c) If, in addition, $v_i(\cdot, \theta_i^{\mathcal{J}})$ is strictly concave on $\mathbf{Z}_i(\theta_i^{\mathcal{J}})$ for all $i \in \mathcal{J}$, then there is a unique local jungle equilibrium at $\boldsymbol{\theta}^{\mathcal{J}}$.

Proof. (a)–(b) We first claim that $\hat{\mathbf{z}} \in \mathbf{Z}_{h(|\mathcal{J}|-1)}(\boldsymbol{\theta}^{\mathcal{J}})$, the constraint set of the weakest individual in the serial dictatorship recursion. Indeed, since $\hat{\mathbf{z}}$ is a jungle equilibrium, it belongs to $\mathbf{Z}(\boldsymbol{\theta}^{\mathcal{J}})$. By the nested construction of the solution sets, we have

$$\mathbf{Z}(\boldsymbol{\theta}^{\mathcal{J}}) \setminus \mathbf{Z}_{h(|\mathcal{J}|-1)}(\boldsymbol{\theta}^{\mathcal{J}}) = \bigcup_{n=1}^{|\mathcal{J}|-1} \mathbf{Y}_{h(n)}(\boldsymbol{\theta}^{\mathcal{J}}).$$

If $\hat{z}$ were not in $Z_{h(|\mathcal{J}|-1)}(\theta^\mathcal{J})$, it would belong to $Y_{h(m)}(\theta^\mathcal{J})$ for some $m < |\mathcal{J}|$. From [Assumption 2](#), there is a weaker individual $h(n)$, with $n > m$, and a feasible trade profile $\tilde{z}$ that strictly increases the payoff of individual $h(m)$ while leaving every individual other than $h(m)$ and $h(n)$ at her payoff under $\hat{z}$. This contradicts $\hat{z}$ being a local jungle equilibrium.

Since $\hat{z} \in Z_{h(|\mathcal{J}|-1)}(\theta^\mathcal{J})$, we have

$$v_{h(n)}(\hat{z}_{h(n)}, \theta_{h(n)}^\mathcal{J}) = \pi_{h(n)}(\theta^\mathcal{J}), \quad \text{for all } n = 1, \dots, |\mathcal{J}| - 1.$$

The weakest individual's payoff from the serial dictatorship is the maximum of her payoff over the same set:

$$\pi_{h(|\mathcal{J}|)}(\theta^\mathcal{J}) = \max \left\{ v_{h(|\mathcal{J}|)}(z_{h(|\mathcal{J}|)}, \theta_{h(|\mathcal{J}|)}^\mathcal{J}) : z \in Z_{h(|\mathcal{J}|-1)}(\theta^\mathcal{J}) \right\}.$$

Since $\hat{z}$ belongs to this constraint set, $v_{h(|\mathcal{J}|)}(\hat{z}_{h(|\mathcal{J}|)}, \theta_{h(|\mathcal{J}|)}^\mathcal{J}) \leq \pi_{h(|\mathcal{J}|)}(\theta^\mathcal{J})$. Equality holds precisely when $\hat{z}$ solves the final optimization problem, that is, when $\hat{z} \in Z_{h(|\mathcal{J}|)}(\theta^\mathcal{J}) = Z^*(\theta^\mathcal{J}, h)$.

(c) From [Corollary 1\(c\)](#), the set $Z^*(\theta^\mathcal{J}, h)$ is a singleton. Starting from the convex set $Z(\theta^\mathcal{J})$, strict concavity of $v_{h(1)}(\cdot, \theta_{h(1)}^\mathcal{J})$ implies that all maximizers at the first step of the recursion have the same trade position for the most powerful individual $h(1)$. The solution set is therefore obtained by fixing that individual's trade position and remains convex. Repeating this argument shows that $Z_{h(|\mathcal{J}|-1)}(\theta^\mathcal{J})$ fixes the trade positions of the first $|\mathcal{J}| - 1$ individuals in the hierarchy. Feasibility then fixes the weakest individual's trade position through $\sum_{i \in \mathcal{J}} z_i = \mathbf{0}$. Hence this set is a singleton and coincides with $Z^*(\theta^\mathcal{J}, h)$. By parts (a)–(b) of the proposition, every local jungle equilibrium belongs to this singleton, while [Proposition 1](#) shows that its unique element is a local jungle equilibrium. This proves uniqueness at $(\theta^\mathcal{J}, h)$. $\square$

B. General Existence of Occupational Equilibria

Fix a nonempty jungle $\mathcal{J}$ satisfying the conditions of [Proposition 3](#). We use the standard mixed-strategy formulation of continuous games ([Glicksberg, 1952](#), Section 2). Individual i 's strategy space is $\Delta(\Theta_i^\mathcal{J})$, the Borel probability measures on her occupational menu, endowed with the topology of weak convergence. A strategy profile is then $\sigma = (\sigma_i) \in \times_{i \in \mathcal{J}} \Delta(\Theta_i^\mathcal{J})$. Individuals draw their occupations independently according to these lotteries, before the hierarchy is drawn and independently of its realization. The realized occupations determine their feasible trade sets and valuations, and the serial dictatorship then determines trades.

Let ν_σ denote the product probability law induced by σ on $\Theta^\mathcal{J}$. Individual i 's payoff in the occupational game with lotteries is

$$\Pi_i(\sigma) \equiv \int_{\Theta^\mathcal{J}} \bar{\pi}_i(\theta^\mathcal{J}) d\nu_\sigma(\theta^\mathcal{J}). \quad (15)$$

The payoff $\bar{\pi}_i$ in [Equation 2](#) already averages over hierarchies at a fixed occupational profile; Π_i also averages over occupational choices. This integral is well defined because $\bar{\pi}_i$ is continuous and bounded on the compact set $\Theta^\mathcal{J}$.

An occupational equilibrium in lotteries is a profile $\sigma^* \in \prod_{i \in \mathcal{J}} \Delta(\Theta_i^{\mathcal{J}})$ such that

$$\Pi_i(\sigma_i^*, \sigma_{-i}^*) \geq \Pi_i(\sigma_i', \sigma_{-i}^*), \quad i \in \mathcal{J}, \quad \sigma_i' \in \Delta(\Theta_i^{\mathcal{J}}). \quad (16)$$

Thus no individual gains by changing her occupational lottery while the others keep theirs fixed. A pure occupation $\theta_i^{\mathcal{J}}$ is represented by the Dirac measure $\delta_{\theta_i^{\mathcal{J}}}$, so pure occupational equilibria are included in this formulation.

The mixed-strategy spaces are compact and convex, and Π_i is continuous in the strategy profile and affine in σ_i . Consequently, the convexity of pure occupational best replies required in [Theorem 1](#) is unnecessary here.

Theorem 4. *Let $\mathcal{J} \neq \emptyset$ and suppose that the conditions of [Proposition 3](#) hold. Then the occupational game with strategy spaces $\Delta(\Theta_i^{\mathcal{J}})$ and payoffs Π_i admits an equilibrium satisfying [Equation 16](#).*

Proof. There are finitely many individuals, each menu $\Theta_i^{\mathcal{J}}$ is a nonempty compact metric space, and each payoff π_i is continuous by [Proposition 3\(b\)](#) and finite averaging over hierarchies. Glicksberg's existence theorem ([Glicksberg, 1952](#), Section 2) therefore applies to this occupational game and yields a profile of lotteries satisfying [Equation 16](#). $\square$