

# **Dynamical Systems and Optimal Control for Economists**

Jorge R. Chávez Fuentes & Marcelo M.  
Gallardo Burga

# Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
1.1	Introduction . . . . .	1
1.2	Preliminaries . . . . .	3
1.3	First order linear differential equations . . . . .	21
1.4	Two particular cases . . . . .	35
<b>2</b>	<b>Nonlinear scalar models</b>	<b>48</b>
2.1	Introduction . . . . .	48
2.2	Maximal solution and continuous dependence . . . . .	49
2.3	Qualitative analysis . . . . .	60
2.4	Bifurcations . . . . .	86
2.5	Numerical solution . . . . .	103
<b>3</b>	<b>Linear Systems</b>	<b>117</b>
3.1	Introduction . . . . .	117
3.2	Matrix exponential . . . . .	124
3.3	Linear systems in the plane . . . . .	139

3.4	Invariant subspaces . . . . .	184
3.5	Non homogeneous linear systems . . . . .	194
<b>4</b>	<b>Nonlinear dynamical systems</b>	<b>216</b>
4.1	Introduction . . . . .	216
4.2	Linearization and Hartman-Grobman Theorem . . . . .	229
4.3	Stable manifolds and stationary saddle solutions . . . . .	268
4.4	Limit cycles and periodic solutions . . . . .	279
<b>5</b>	<b>Calculus of Variations</b>	<b>312</b>
5.1	Introduction . . . . .	312
5.2	The $\mathcal{P}_v$ problem . . . . .	318
5.3	The Euler-Lagrange Equation . . . . .	324
5.4	Sufficient Conditions and Autonomous Equation .	332
<b>6</b>	<b>Optimal Control Theory</b>	<b>342</b>
6.1	Introduction . . . . .	342
6.2	Maximum Principle of Pontryagin . . . . .	348
6.3	Mangasarian and Arrow conditions . . . . .	369
6.4	Final state conditions . . . . .	381
6.5	Infinite horizon . . . . .	409
<b>A</b>	<b>Topology in vector normed spaces</b>	<b>469</b>
<b>B</b>	<b>Calculus and Real Analysis</b>	<b>481</b>